EXERCISES IN ALGEBRA

ROBBINS AND SOMERVILLE

THORNTON SMITH
2064 DUNWOODY ST.
ATLANTA

EXERCISES IN ALGEBRA

BY

EDWARD R. ROBBINS

AND

FREDERICK H. SOMERVILLE

WILLIAM PENN CHARTER SCHOOL, PHILADELPHIA

NEW YORK ·:· CINCINNATI ·:· CHICAGO

AMERICAN BOOK COMPANY

Copyright, 1904, by

EDWARD R. ROBBINS and FREDERICK H. SOMERVILLE.

Entered at Stationers' Hall, London.

PREFACE

THE present-day teacher of Algebra has little time for the selection, from proper sources, much less for the making, of suitable examples often necessary to supplement those contained in the standard text-books. This little book is designed to meet the requirements of those teachers who feel such extra assignments in Algebra essential to thorough familiarity with its processes. The aim has been to provide, as compactly as possible, a series of exercises that conform in arrangement with the order of the leading text-books, and that both in degree of difficulty and in scope shall include the work prescribed by high schools and academies, as well as university and college entrance requirements.

The plan has been to avoid all examples of more than average difficulty, and to lay particular stress upon those subjects that stand so clearly as the foundations of later work. With this in view, much emphasis has been given to the subjects of Factoring, Fractions, Exponents, Equations, and Logarithms. There has been constant effort to present abundant drill in topics in the mastering of which students seem to have the greatest difficulties.

Not only must the pupil who is to master the science solve a multitude of exercises differing in degree rather than in

kind, but he must also be taught to select appropriate methods for each of a miscellaneous collection, differing in kind rather than in degree. To this end the book is generously provided with reviews, and the pupil is obliged to discriminate among different kinds of examples — a phase of mathematical training which will be invaluable to him in subsequent study. Furthermore, the reviews are so arranged that they could, if desired, be used independently of the other exercises in final review, or in final preparation for college examinations.

The authors will welcome and will appreciate any suggestions or corrections from other teachers of Algebra.

EDWARD R. ROBBINS.
FREDERICK H. SOMERVILLE.

PHILADELPHIA, PA.

CONTENTS

	PAGE
Substitution	7
Addition	9
Subtraction	10
Use of the Parenthesis	13
Review	16
Multiplication	17
Division	18
Multiplication by Inspection	20
Division by Inspection	23
Use of the Parenthesis with Multiplication	25
Simple Equations	26
Problems in Simple Equations	27
Review	29
Factoring	31
Review	38
Highest Common Factor and Lowest Common Multiple	40
Fractions:	
Transformations	42
Addition and Subtraction	45
Multiplication and Division	48
Complex Fractions	50
Fractional Equations:	
Numerical Equations	55
Literal Equations	59
Simultaneous Equations:	
Numerical Equations	61
Literal Equations	64
Three or More Unknown Quantities	66
Problems in Simultaneous Equations	68

PAGE

Involution and Evolution:
- Monomials 71
- Involution — Binomials 72
- Evolution — Square Root 72
- Evolution — Cube Root 74
- Evolution — Numerical 74

Review 76

Exponents:
- Transformations 81
- Miscellaneous Applications 88

Radicals:
- Transformations 93
- Miscellaneous Applications 99

Imaginaries 103

General Review 107

Quadratic Equations:
- Numerical Quadratics 116
- Literal Quadratics 119
- Equations in Quadratic Form 121

Simultaneous Quadratics 123

Properties of Quadratics 126

Ratio and Proportion 129

Variation 132

Arithmetical Progression 134

Geometrical Progression 138

Permutations and Combinations 142

Binomial Theorem 145

Logarithms 148

General Review 156

$3a + c + d$

SUBSTITUTION

Exercise 1

Find the numerical value of the following:

When $a = 1,\ b = 2,\ c = 3,\ d = 4$.

1. $a + b$.
2. $a + b + c$.
3. $a + 2b + 3c$.
4. $a + 3b - d$.
5. $2a + 4b - 2d$.
6. $6a - b - d$.
7. $10c - 5b + 2d$.
8. $3a + c + d$.
9. $3a + (c + d)$.
10. $7b - (c + d)$.
11. $a + ab$.
12. $3ab - c$.
13. $12a - 3c + cd$.
14. $3a + b(a + c)$.
15. $ab + a(2b - a)$.
16. $4abc - 3(c + d)$.
17. $a(a + b + c)$.
18. $ab(a + b + c)$.
19. $ab(d - a) + bc(d - b)$.
20. $a + ab(b + c) - c(3d - 3c)$.
21. $c^2 + ab$.
22. $b^3 + c^3 + d^2$.
23. $2a^2b^2 + 3b^2 - c^2 + d$.
24. $b^2c^2 + bc^2 + d$.
25. $ab^2d - a(b^2 + c)$.
26. $25d - ab^2(a + b + c)$.
27. $a + (a + b)^2$.
28. $4bcd + (2b + c)^2$.
29. $3(a + 2)^2 - 2(b^3 - 1) + 3a^2bc^2$.
30. $2(a + b)^3 - 3(d - b)^2 - bc(c + d)$.

When $a = 5,\ b = 4,\ c = 3,\ d = 2,\ e = 9.$

31. $\sqrt{b} + \sqrt{e}.$

32. $\sqrt{4\,e} - \sqrt{2\,d}.$

33. $\sqrt{5\,ab} + \sqrt{4\,b}.$

34. $\sqrt{3\,ce} - \sqrt{3\,bc}.$

35. $\sqrt{abc + b}.$

36. $\sqrt{abc - 3\,c - d}.$

37. $\sqrt{a^2 + b^2 + 8}.$

38. $\sqrt{2\,c^2 + 3\,d^2 - a}.$

39. $a + b\sqrt{bcd + 1}.$

40. $(a + b)\sqrt{bcd + 1}.$

41. $(ad - e)\sqrt{3\,ac - e}.$

42. $(a + \sqrt{e}) - (e - \sqrt{b}).$

When $a = 4,\ b = 5,\ m = 6,\ n = 10.$

43. $\dfrac{3\,a}{m}.$

44. $\dfrac{m + n}{2\,a}.$

45. $\dfrac{2(n - m)}{a}.$

46. $\dfrac{5(m + n)}{ab}.$

47. $\dfrac{m + n - a}{m}.$

48. $\dfrac{b(n - m)}{2\,n}.$

49. $\dfrac{3\,a + 2\,b}{n - 1}.$

50. $\dfrac{3\,a + (n - m)}{2(n - a)}.$

51. $\dfrac{a}{m} + \dfrac{a}{n} + \dfrac{m}{b}.$

52. $\dfrac{a + m}{b} + \dfrac{a + m}{n} - \dfrac{m + n}{2\,a}.$

53. $\left(\dfrac{\sqrt{2\,ab + am}}{m + n}\right) + \sqrt{n - m}.$

54. $\dfrac{n}{\sqrt{n - m}} - \dfrac{6\,m}{\sqrt{4\,a}} + \sqrt{3\,n - b}.$

When $x = \frac{1}{2},\ y = \frac{2}{3},\ z = \frac{1}{4}.$

55. $x + y + z.$

56. $2\,x - y - z.$

57. $x\,(x + y).$

58. $y\,(2\,x - z).$

59. $(x + y)(y - z).$

60. $xy(x + y - z).$

61. $(x + 1)^2 - (y + 1)^2 + (z + 1)^2.$

62. $x + y\left(\dfrac{y + z}{x}\right) - (x + z)^2.$

When $a=4,\ b=5,\ m=2,\ n=3$.

63. a^m.

64. a^m+b^n.

65. a^n-b^m.

66. $2\,a^n+3\,a^m b^n$.

67. $(a+b)^m$.

68. $(a+b)^n-(a+b)^m$.

69. $(2\,b-a)^n-(3\,b-a)^m$.

70. $(a+b-7)^m$.

71. $(a^m+b^m)^n$.

72. $a^n+a^m(b-a)^n$.

ADDITION

Exercise 2

1. Find the sum of $a+3\,b+c$, $2\,a+7\,b+2\,c$, and $3\,a+2\,b+c$.

2. Find the sum of $4\,a+3\,b-c$, $2\,a+2\,b+4\,c$, and $a-3\,b-2\,c$.

3. Find the sum of $10\,a-3\,b-2\,c-d$, $-2\,a+4\,b+c+3\,d$, and $c-3\,d+a$.

4. Add $3\,a+2\,b-3\,c$, $12\,a-4\,b-7\,c$, and $4\,a-8\,b+9\,c$.

5. Add $3\,x^3+2\,x^2-2\,x+11$, $4\,x^3-2\,x^2+3\,x-8$, $4\,x^3-2\,x^2+x-2$, $-12\,x^3+x^2-x-1$, and $2\,x^3+x^2-x+13$.

6. Collect $5\,a-3\,c+4\,f-m+2\,b-d+4\,c-2\,a-3\,f+c+2\,m+d$.

7. Collect $2\,ab+3\,bc+4\,cd-2\,bc+3\,ab-3\,cd-4\,ab-2\,bc-2\,cd$.

8. Collect $2\,abcd+3\,bx+m-3\,abcd+2\,bx-3\,m+6\,abcd-3\,bx+3\,m$.

9. Collect $a^3+3\,a^2+3\,a$, a^2+a+1, and $2\,a+2$.

10. Collect $a^3+a^2b+ab^2$, $3\,a^3+a^2b+2\,b^3$, and $4\,ab^2-3\,b^3$.

11. Arrange in descending powers of x and collect $x^3 - 2 + 3x^2 - x$, $-x + 3x^2 - 2x^3$, $-4x^2 + 5x^3 - x + 10$, $-x + 4x^2 - 2 + x^3$, and $x^3 + x^2 + x + 1$.

12. Arrange in ascending powers of a and add $a^2 - 4a^3 + 2a - 8$, $a^4 + a - 3a^2 + 16$, $a^3 + a^2 - 2 + 3a$, and $-4a^2 + 6a^3 + 10a - 3$.

13. Find the sum of $x^3 - 1 + 3x - x^2 + 2x - 3x^3 + 4 + x^2 - 8x + 2x^3 - 4x^2 + 10 - x^2$, and arrange the answer in descending powers of x.

14. Add $3(a+b) + 4(c+d) - 5(m+n)$, $2(a+b) - 2(c+d) + 6(m+n)$, and $-4(a+b) - (c+d) - (m+n)$.

15.	16.	17.	18.
ac	ac	mx	my
bc	c	$5x$	$-3y$
$(a+b)c$	$(a+1)c$	$(\quad)x$	$(\quad)y$

19. Add $ab + cd$ and $mb + nd$.

20. Add $ax + by$ and $cx + dy$.

21. Add $2ab + 3c + d$ and $3b + ac + nd$.

22. Add $3ab + 2ac + n$, $3b - 3ac + mn$, and $2mb + 4bc - pn$.

23. Add $(a+b)c + (b+d)c$.

24. Add $(m+n)x + (m-n)x - mx$.

25. Add $(2+a)x^2 + (3a-4)x^2 + (a-1)x^2 - 4ax^2$.

SUBTRACTION

Exercise 3

Subtract:

1.	2.	3.	4.	5.	6.	7.	8.
$10a$	$10a$	$-4a$	$-3a$	$15a^2$	$4ab$	$3ab$	$2am$
$3a$	$-4a$	$3a$	$-a$	$-2a^2$	$3ab$	$7ab$	$-6am$

9.	10.	11.	12.	13.
$3a+2b$	$14a+3c$	$16a-2c$	$12ab+16$	$-5a-4c$
$a+b$	$7a-c$	$3a+5c$	$3ab-10$	$-2a-5c$

14.	15.	16.	17.
a^2-10ab	$2a^2+12am$	$5ab$	$16am+mn$
a^2+3ab	$3a^2+3am$	$4ab+cd$	$3am$

18.	19.	20.
$4a^2c+3ab+10$	$15a^2$	$10a^3+3a^2+6a+7$
$-2ab$	$10a^2+4a+4$	a^2-4

21. Take $a+b+c$ from $4a+3b+2c$.

22. Take $5a-b+c$ from $6a+3b-4c$.

23. Take $2a+3b-4c$ from $4a+3b+8c$.

24. Take $3a+10b-14c$ from $4a+10b-14c$.

25. Take $a^2+3ab+12$ from $5a^2-8ab+16$.

26. Take $4a^2+3ab+2ab^2-10$ from $8a^2-16ab-3ab^2-1$.

27. Take $3a^2+3a+1$ from $4a^3+5a^2-6a-3$.

28. From $3a^4-2a^3+6a^2+2a-3$ take $2a^4-a^3-4a^2-a+1$.

29. From $a^4-4a^3b+6a^2b^2-4ab^3+b^4$ take $a^3b+3a^2b^2+ab^3$.

30. From $-3a^2+8a+36$ take $a^2-2a-18$.

31. From $-16a^3-8a^2+4a-5$ take $-2a^3+a^2-a+1$.

32. Given a minuend a^3+4a^2-3a+2 and a subtrahend a^3+3a^2-4a+1, find the difference.

33. Given a subtrahend $-3ac$, a minuend $8ac$, find difference.

34. Given a minuend $3\,abc$, a difference $4\,abc$, find subtrahend.

35. Given a minuend axy, a difference $-axy$, find subtrahend.

36. What must be added to $16\,ac$ to make $-3\,ac$?

37. What must be added to $75\,m$ to make $31\,m$?

38. What must be added to $-3\,a^2$ to make 0?

39. $36\,a^2$ is added to what expression to make $82\,a^2$?

40. What expression added to $4\,a+2\,c$ will give $5\,a+8\,c^2$?

41. Given a subtrahend $4\,a^4+3\,a^3-2\,a^2+a-7$ and a difference $-3\,a^3+2\,a^2-a$, find the minuend.

42. Given a minuend $8\,m^2-3\,mn+2\,n^2+18$ and a difference $2\,m^2+2\,mn-n^2+16$, find the subtrahend.

43. Take the sum of $4\,m^2+2\,mn-n^2$ and $-3\,m^2-mn+2\,n^2$ from the sum of $3\,m^2+12\,mn+10\,n^2$ and $-2\,m^2-11\,mn-11\,n^2$.

44. Subtract $x^2-3\,x^5+x^3-2-3\,x$ from $x+x^5-3+2\,x^4-x^2-x^3$, and arrange the result in ascending powers of x.

45.	46.	47.	48.	49.	50.
ac	am	x	$2\,ax$	$5\,abx$	am
bc	m	cx	$-3\,cx$	$3\,cdx$	$-2\,cm$
$(a-b)\,c$	$(a-1)\,m$	$(1-c)\,x$	$(2\,a+3\,c)\,x$		

51.	52.	53.	54.	55.
$3\,mn$	$3\,ax+2$	$ax+by$	$x+y$	$3\,ax+y$
$-2\,pn$	$cx-1$	$cx+dy$	$cx-dy$	$x-cy$

56. Take $2\,bc-3\,ad$ from $3\,ac+12\,bd$.

57. From $ax+3\,cy+8\,dz$ take $bx-2\,dy-az$.

58. From $3\,ax+2\,ay+10$ take $2\,x-y+1$.

USE OF THE PARENTHESIS

Exercise 4

Simplify:

1. $a+(3a+2)$.
2. $4a+(2a-10)$.
3. $5a+(3a-2)+1$.
4. $2x+(4x-y)+(3x-2)+y$.
5. $(5m+7)+(3m-2)$.
6. $2c+(3c+4)-(c+2)$.
7. $5m-(m+1)-(2m-1)$.
8. $6y-(4y+3z)-(2y-4z)$.
9. $3a+[a+(4a+3)]$.
10. $8a+[a-(2a-1)]$.
11. $5m-\{3m+(2m-1)\}$.
12. $2a+\{-3+(2a-1)\}$.
13. $11a-\{4a+(10a-6)\}$.
14. $-3a+\{2a-(-a+1)\}$.
15. $-2a-[-a-\{-a+1\}]$.
16. $13a+\{-4a-(10a-[a-1])\}$.
17. $a+[a-\{a-(a-1)\}]$.
18. $a-[-a-\{-a-(-a-1)\}]$.
19. $5a-[2a-(a+1)-\{3a-(a+1)-1\}-1]$.
20. $6a-[(3a+1)+\{3a-6-(a+2)-3a\}-a]$.
21. $(5a-1)-[\{(3a-1)-10a+5\}-a]-\{-(a-2)\}$.
22. $(3+\overline{a-1})-[-a+(a-\overline{2a-3})-3a]-\{a-\overline{1+a}\}$.

23. $(4a^2-[3a^2-(a+2)-4]-\{a^2+\overline{3a-(a-1)}\})-a.$

24. $1-(-1)+(-1)-\{1-[1-(1-\overline{1+a})+a]-a\}.$

25. $1-\{-[-(-1)-1]-1\}-(-\{-[-(-1)]\}).$

26. $3x-\{a-(2a-[3a-(5a-[7a-(8a-x)])])\}.$

27. $-\overline{m-[3n+\{2p-3m-(m-n)\}+n-\{3m-(m+n)\}]}.$

28. $(4a-\overline{a+2})-\{a-\overline{a+(a-2)}-2\}-a-\overline{a+1}.$

Simplify and find numerical values of the following when $a=5$, $b=3$, $c=-1$, $d=-2$, and $x=0$:

29. $a+b+c+d+x.$

30. $2a+c-3b-d.$

31. $a+2c-(d+2)+2(c+1).$

32. $a+c-d-3b+x.$

33. $4b-d+ab-ax.$

34. $2a-(c+d)-(a+d).$

35. $4abx+(a-d)-(b-c).$

36. $c+3x-2(a+2x)+4a.$

37. $a+2b-[3a+c-\{2a-(x-c)\}].$

38. $a+3c-\{b+2x-(a-c)\}+2d.$

39. $3a-[5b-(ab+d)-\overline{a-d}]-(ab+x).$

40. $a^2-5x-[3ab+4b^2-(abx+d)].$

41. $ax-[-b+(a-d)^2+a]-2b.$

42. $(a+c)^2+(a-c)^2-[ab+(2b+x)^2].$

43. $(a+c)x-(abx+a)^2-(d+c).$

44. $(a+x)b-[\sqrt{2a+c}-(d+c)].$

45. $x-[c+(a+d)^2-2b]^2+(a-\sqrt{ab-c})^2.$

46. $ab^2-\{3(b+c)^2-[-c+2a(4d+b^2)]-[bcx+\sqrt{3a-c}]\}.$

Exercise 5

1. Insert the last two terms of $a+b-c$ in a parenthesis preceded by a plus sign.

2. Insert the last two terms of $a-b-c$ in a parenthesis preceded by a minus sign.

Insert the last three terms of each of the following expressions in a parenthesis preceded by a minus sign:

3. $a-b-c-d$.
4. $a-b+c+d$.
5. $a+b-c-d$.
6. $a+b+c+d$.
7. $3a^4-2a^3-a+2$.
8. $4a^4-a^3+a^2-3a+1$.
9. $6a^2-a^3-10a^4+a^5-a^6$.

Collect the coefficients of a, of b, and of c, in each of the following:

10. $3a-2b+c-ma-nb-pc$.
11. $5a-3mb+2nc-2a-6nb-3c-ma$.
12. $10a-4b-2c-ma-nb+pc$.
13. $-3a-4c-pb-na-mc-b$.
14. $-c-a+4b-3c+da-10b$.
15. $-4a-3mb-10c-b-ma-nb+pc-da$.

Collect the coefficients of like terms in each of the following:

16. $4x^3-2x+3x^2-3x+4x^3-x^2$.
17. $-3x^2-2x+x^4-3x^2+2x-x^4$.
18. $ax^3-bx^2-cx-cx^3-dx+x^2$.
19. $x^3+x^2-ax-cx^3+3x^2-bx$.
20. $-ax^3-x^2-x-4x^3-ax^2-px-cx^3$.
21. $-px^3-cx+3x^2-mx-nx^3-mx^2-abx+3dx^2-cx^3$.

REVIEW

Exercise 6

Find the numerical value of:

1. $(x+2)^2-(x+1)-4$ when $x=3$.

2. $(x-2)(x+5)-2(x+1)^2$ when $x=4$.

3. $(a+b)^2-2(a-1)(b-1)-ab^2$ when $a=5$ and $b=3$.

4. $\sqrt{x^2+m^2}-(n-my)^2$ when $x=4$, $y=0$, $m=3$, $n=2$.

5. $2a-[a-(3a-2b)]$ when $a=3$ and $b=2$.

6. $3a-[4b+2m-\overline{3c+(a-b)}-2a]$ when $a=4$, $b=3$, $c=-2$, and $m=1$.

7. Subtract the sum of a^3+a^2-a and $2a^2+2a-1$ from the sum of $3a^3-a^2-a+1$ and $2a^2+4a-3$.

8. Take m^3-3m^2 from the sum of $2m^4-m^3+2$ and m^2-3m+1.

9. From the sum of a^3+a^2-a and a^3+4 take the sum of $4a^3-2a^2+17$ and $a-3$.

10. If $A=x^2+x+1$, $B=2x^2-x+2$, and $C=x^2-3x+6$, find the value of $A+B-C$.

11. With values for A, B, and C equal to those in Example 10, find the value of $A-B+C$.

12. To what expression must you add $5a^2$ to make 0?

13. What expression added to x^2+x-1 will make 0?

14. $-x^2-x+3$ is subtracted from $2x^2-3x+4$, and the remainder is added to $-3x^2+2x-14$. What expression results?

MULTIPLICATION

Exercise 7

Multiply:

1. $a+1$ by $a+2$.
2. $a-2$ by $a-5$.
3. $a+4$ by $a-3$.
4. a^2+1 by a^2+2.
5. a^2+3 by $2a^2-5$.
6. a^2+a+1 by $a+1$.
7. a^2+a+1 by $a-1$.
8. a^3+3a^2+3a+1 by $a+1$.
9. x^4-2x^2+1 by x^2+1.
10. $4x^2+4x+1$ by $2x+1$.
11. a^2-3a-2 by a^2-a+1.
12. a^3-3a^2+3a-1 by a^2-2a+1.
13. $12y^2-6y+2$ by y^2-3y+1.
14. $a^4-4a^3+6a^2-4a+1$ by a^3-3a^2+3a-1.
15. $5a^3-2a^2+3a-1$ by $3a^2+a-1$.
16. $m^4-2m^3+3m^2-2m+1$ by $m^4+2m^3+3m^2+2m+1$.
17. m^3-m^2+3m-5 by m^3+m^2+3m+5.
18. $a^2+b^2-2ab-4b+4a+4$ by $a-b+2$.
19. x^2+x^3+1+x by $1+x^2+x$.
20. $1-7x^2+x^3+5x$ by $-4x+1+2x^2$.
21. $1-4a^3-4a+6a^2+a^4$ by $3a-1+a^3-3a^2$.
22. x^5-2x^2+3 by $1-x^2+x$.
23. $a^2+2ab+b^2+m^2$ by $a^2+2ab+b^2-m^2$.
24. a^3-a^2+a-1 by $1+a+a^2+a^3$.
25. $5x^3-2x^2+x^4-3$ by $-3x^2+2x^3-2+x^4$.
26. $a^2+2ab+b^2$ by $a^3-3a^2b+3ab^2-b^3$.
27. $a^4-4ab^3+b^4+6a^2b^2-4a^3b$ by b^2+a^2-2ab.

28. $3a^3b - 2a^2b^2 + 4ab^3$ by $2a^2b + b^3 - ab^2$.

29. $5a^4m + 4a^2m^3 - 3a^3m^2 - am^4$ by $2a^2m - 3am^2$.

30. $(a+1)(a+2)(a+3)$.

31. $(a+2)(a-2)(a+1)$.

32. $(a+1)(a-1)(a+3)(a-3)$.

33. $(x+2)(x+2)(x-2)(x-2)$.

34. $(m^2+m+1)(m^2-m+1)(m^2-m-1)$.

35. $(2x^2+x+1)(2x^2+3x+1)(x^2-2x+1)$.

36. $(3x^2-5x+3)(x-4)(x^2-x+2)$.

37. $(a+b)+2$ by $(a+b)+3$.

38. $(a+2)+3x$ by $(a+2)+4x$.

39. $(a+c)-4$ by $(a+c)+6$.

40. $(a+b)+(c+d)$ by $(a+b)+2(c+d)$.

DIVISION

Exercise 8

Divide:

1. a^2+4a by a.
2. $3a^3+6a^2$ by $3a^2$.
3. $a^4-a^3+a^2$ by a^2.
4. $a^4-a^3+a^2$ by $-a^2$.
5. $5a+10a^2+15a^3$ by $5a$.
6. $12m^3-9m^2+3m$ by $3m$.
7. $-m^3+m^2+4m$ by $-m$.
8. $15a^2b^2-75ab$ by $-5ab$.
9. $27a^3b^6+36a^2b^5-18ab^4$ by $-9ab^3$.
10. a^2+4a+3 by $a+1$.
11. $1+8a+15a^2$ by $1+3a$.
12. a^3+3a^2+3a+1 by $a+1$.

13. $8a^3+12a^2+6a+1$ by $2a+1$.

14. $16-32a+24a^2-8a^3+a^4$ by $2-a$.

15. $a^4-4a^3+6a^2-4a+1$ by a^2-2a+1.

16. $m^3+5m^2n+5mn^2+n^3$ by $m^2+4mn+n^2$.

17. $4m^4-9m^2+6m-1$ by $2m^2+3m-1$.

18. $a^5-10a^4+40a^3-80a^2+80a-32$ by a^2-4a+4.

19. $m^4-3m^3-36m^2-71m-21$ by m^2-8m-3.

20. $4a^4-15a^3c+26a^2c^2-23ac^3+8c^4$ by $4a^2-7ac+8c^2$.

21. $m^6-6m^4+5m^2-1$ by m^3+2m^2-m-1.

22. $a^5+12a^2-48+52a-17a^3$ by $a-2+a^2$.

23. $m^4+4m^2n^2+16n^4$ by $m^2+2mn+4n^2$.

24. $18c^4+82c^2+40-67c-45c^3$ by $5-4c+3c^2$.

25. $4x^2+13x-6x^3+6+x^5-2x^4$ by $1+x^3+3x+3x^2$.

26. $25m-10m^3+15+14m^4-41m^2$ by $-5m-3+7m^2$.

27. $-14c^4d+12c^5+10c^3d^2-c^2d^3-8cd^4+4d^5$ by $2d^3-3cd^2-4c^2d+6c^3$.

28. $m^2-n^2-p^2-2pn$ by $m-n-p$.

29. $c^3+d^3+m^3-3cdm$ by $c+d+m$.

30. $a^3+b^3+c^3-3abc$ by $a^2+b^2+c^2-ac-bc-ab$.

31. x^2-y^2 by $x-y$.

32. x^3-y^3 by $x-y$.

33. x^4-y^4 by $x-y$.

34. x^4-y^4 by $x+y$.

35. x^3-27 by $x-3$.

36. x^4-16 by $x+2$.

37. $32-m^5$ by $2-m$.

38. $64+27a^3$ by $4+3a$.

39. $16-81a^4$ by $2+3a$.

40. $125c^3-8$ by $5c-2$.

41. $216m^3-27$ by $6m-3$.

42. $81x^4-16y^4$ by $3x+2y$.

43. $9m^4-49c^8$ by $3m^2+7c^4$.

44. $27a^6+64b^9$ by $3a^2+4b^3$.

MULTIPLICATION BY INSPECTION

Exercise 9

The square of the sum of two quantities.

Expand by inspection:

1. $(a+b)^2$.
2. $(a+c)^2$.
3. $(a+m)^2$.
4. $(a+2)^2$.
5. $(a+3)^2$.
6. $(a+5)^2$.
7. $(a+10)^2$.
8. $(2a+b)^2$.
9. $(3a+c)^2$.
10. $(2a+3x)^2$.
11. $(7ax+3)^2$.
12. $(8a^2+3b^2)^2$.

The square of the difference of two quantities.

Expand by inspection:

13. $(a-b)^2$.
14. $(a-m)^2$.
15. $(a-4)^2$.
16. $(a-6)^2$.
17. $(a-2c)^2$.
18. $(3a-2)^2$.
19. $(ab-c)^2$.
20. $(a^2-ab)^2$.
21. $(2a-3cd)^2$.
22. $(3a^2-2ab)^2$.
23. $(4ax-xy)^2$.
24. $(9a^2-10c^2)^2$.

The product of the sum and difference of two quantities.

Multiply by inspection:

25. $(a+b)(a-b)$.
26. $(a+c)(a-c)$.
27. $(a+2)(a-2)$.
28. $(a-4)(a+4)$.
29. $(a^2-3)(a^2+3)$.
30. $(2ab+1)(2ab-1)$.
31. $(4a+3)(4a-3)$.
32. $(5a^2-2c)(5a^2+2c)$.
33. $(6a^2-7ab)(6a^2+7ab)$.
34. $(5xy-3xz)(5xy+3xz)$.

Perform the indicated multiplications:

35. $(a-2c)^2$.
36. $(a+3x)^2$.
37. $(a^2-2)(a^2+2)$.
38. $(a^4-1)(a^4+1)$.
39. $(a^3+3)^2$.
40. $(ab+5c)^2$.
41. $(abc-2)^2$.
42. $(am+mn)(am-mn)$.
43. $(4ac+7)(4ac-7)$.
44. $(6m^2-3)^2$.
45. $(8m+5)(8m-5)$.
46. $(3c^5+5)(3c^5-5)$.

The product of the sum and difference of two quantities obtained by grouping terms.

Multiply by inspection:

47. $[(a+b)+c][(a+b)-c]$.
48. $[(a+x)+3][(a+x)-3]$.
49. $[(a-2)+c][(a-2)-c]$.
50. $[(a^2+1)+a][(a^2+1)-a]$.
51. $[c+(a+b)][c-(a+b)]$.
52. $[m+(n-p)][m-(n-p)]$.
53. $(a+b+c)(a+b-c)$.
54. $(a+x+y)(a+x-y)$.
55. $(a^2+a+1)(a^2+a-1)$.
56. $(a-x+y)(a-x-y)$.
57. $(a-x+y)(a+x+y)$.
58. $(c-d+3)(c+d+3)$.
59. $(a+m-n)(a-m+n)$.
60. $(c+d-3)(c-d+3)$.
61. $(x^2-1+x)(x^2-1-x)$.
62. $(x^2+x-1)(x^2-x+1)$.
63. $[(a+b)+(c+d)][(a+b)-(c+d)]$.
64. $[(m+n)-(x-y)][(m+n)+(x-y)]$.
65. $(a-m+n-1)(a+m+n+1)$.
66. $[m^3-3m^2-m-3][m^3+3m^2+m-3]$.

The square of any polynomial.

Expand:

67. $(a+b+c)^2$.
68. $(a+b-2c)^2$.
69. $(a+b-c-d)^2$.
70. $(a+2b-3c+d)^2$.
71. $(a^2+a-2)^2$.
72. $(2x^2-3xy-5y^2)^2$.
73. $(a^3+2a^2-2a-3)^2$.
74. $(2x^3-x^2y+3xy^2-2y^3)^2$.

The product of the forms $(x \pm a)(x \pm b)$.

75. $(x+2)(x+3)$.
76. $(x+3)(x+5)$.
77. $(x+10)(x+7)$.
78. $(x+3)(x-2)$.
79. $(x+4)(x-5)$.
80. $(x-4)(x+1)$.
81. $(x+3)(x-16)$.
82. $(x-4)(x+12)$.
83. $(x^2-3)(x^2+7)$.
84. $(x^2+4)(x^2-13)$.
85. $(x^3+2)(x^3+19)$.
86. $(x^2y^2+7)(x^2y^2-2)$.
87. $(ax+13)(ax-12)$.
88. $(a^2cx-11)(a^2cx+3)$.

The product of the forms $(ax \pm b)(cx \pm d)$.

89. $(2x+1)(3x+5)$.
90. $(3x+2)(4x+3)$.
91. $(3a-7)(2a+3)$.
92. $(3m+8x)(8m-11x)$.
93. $(4a+3c)(3a+2c)$.
94. $(2c^2-7)(3c^2-11)$.
95. $(a^2-3a)(4a^2+2a)$.
96. $(4ac+7m)(3ac-6m)$.
97. $(11x^2-3y^2)(5x^2+y^2)$.
98. $(mn-13y)(5mn+y)$.
99. $(11m^2-3n)(3m^2+11n)$.
100. $(6cd-3dm)(4cd+8dm)$.

DIVISION BY INSPECTION

Exercise 10

The difference of two squares.

Divide by inspection:

1. $a^2 - b^2 \div a - b$.
2. $a^2 - 9 \div a + 3$.
3. $a^2 - 16 \div a - 4$.
4. $4a^2 - 1 \div 2a + 1$.
5. $16a^2 - 9 \div 4a - 3$.
6. $25m^2 - 81 \div 5m + 9$.
7. $81m^4 - 49 \div 9m^2 - 7$.
8. $64a^2 - 36 \div 8a + 6$.
9. $169a^2 - 9 \div 13a + 3$.
10. $4m^4 - 225 \div 2m^2 - 15$.

The difference of two cubes.

Divide by inspection:

11. $m^3 - n^3 \div m - n$.
12. $a^3 - b^3 \div a - b$.
13. $m^3 - 1 \div m - 1$.
14. $c^3 - 8 \div c - 2$.
15. $27 - x^3 \div 3 - x$.
16. $1 - 64a^3 \div 1 - 4a$.
17. $8a^6 - 1 \div 2a - 1$.
18. $125m^3 - 27 \div 5m - 3$.
19. $1 - 512m^6 \div 1 - 8m^2$.
20. $343c^3 - d^6 \div 7c - d^2$.
21. $125 - 216m^9 \div 5 - 6m^3$.
22. $1000 - m^3n^3 \div 10 - mn$.

The sum of two cubes.

Divide by inspection:

23. $m^3 + n^3 \div m + n$.
24. $8 + c^3 \div 2 + c$.
25. $27m^3 + 1 \div 3m + 1$.
26. $64a^3 + 27 \div 4m + 3$.
27. $125a^3 + 8 \div 5a + 2$.
28. $a^3m^3 + 27 \div am + 3$.

29. $a^6m^6 + 64 \div a^2m^2 + 4$.

30. $8\,a^3 + 343 \div 2\,a + 7$.

31. $216\,a^3 + 512 \div 6\,a + 8$.

32. $1000\,a^3 + 729 \div 10\,a + 9$.

The sum or difference of like powers.

Divide by inspection:

33. $a^4 - b^4 \div a - b$.

34. $a^5 - b^5 \div a - b$.

35. $a^4 - b^4 \div a + b$.

36. $a^5 + b^5 \div a + b$.

37. $a^6 - b^6 \div a - b$.

38. $a^4 - 16 \div a - 2$.

39. $a^5 + 32 \div a + 2$.

40. $16\,a^4 - 1 \div 2\,a + 1$.

41. $32\,a^5 + 1 \div 2\,a + 1$.

42. $81 - 16\,a^4 \div 3 - 2\,a$.

43. $64 - m^6 \div 2 + m$.

44. $32 + 243\,a^5 \div 2 + 3\,a$.

Give the binomial divisors possible for each of the following:

45. $a^3 - 8$.

46. $a^4 - 16$.

47. $a^5 + 32$.

48. $a^4 - 81$.

49. $a^6 - 64$.

50. $a^8 - b^4$.

51. $9a^4 - 16$.

52. $81 - a^6$.

53. $16 - a^4$.

54. $a^8 - 9$.

55. $a^6 + b^6$.

56. $a^{12} - b^{12}$.

Give the quotients of the following:

57. $a^2 - 16 \div a + 4$.

58. $a^4 - 16 \div a - 2$.

59. $4\,a^4 - 25 \div 2\,a^2 + 5$.

60. $81\,a^4 - 36\,c^2 \div 9\,a^2 - 6\,c$.

61. $100\,m^4 - 1 \div 10\,m^2 - 1$.

62. $125\,a^3 - 8 \div 5\,a - 2$.

63. $100\,c^4 - a^8 \div 10\,c^2 + a^4$.

64. $36\,m^4 - 1 \div 6\,m^2 + 1$.

65. $64\,a^6 - 27\,c^{12} \div 4\,a^2 - 3\,c^4$.

66. $1 - 81\,a^8 \div 1 + 3\,a^2$.

67. $100\,(a+1)^2 - 9 \div 10\,(a+1) + 3$.

68. $27\,(a+1)^3 - 8\,c^3 \div 3\,(a+1) - 2\,c$.

USE OF THE PARENTHESIS WITH MULTIPLICATION

Exercise 11

Simplify:

1. $a+(a+1)$.
2. $a+(2a-3)$.
3. $a+2(a+1)$.
4. $a+2(a-4)$.
5. $a^2+(a+1)^2$.
6. $a+2(a-1)$.
7. $(a+2)(a-1)$.
8. $5a+3(a+2)$.
9. $(a+4)(a-2)$.
10. $2a+3(a-3)$.
11. $(a+1)(a+2)-a^2$.
12. $(a+2)^2+(a+1)^2$.
13. $(a+4)^2-(a+2)^2$.
14. $7a^2-2(a^2+1)$.
15. $2(a+1)^2-a(a+1)$.
16. $4(a-1)^2-3(a+1)^2$.
17. $4a+(a+1)(a-1)-(a-1)^2$.
18. $2(a+1)(a+2)-(a+1)(a-2)$.
19. $(a+1)^2-(a+1)(a-1)-2a$.
20. $3a^2+(a+3)(a-1)-(a-3)(a+1)-2(a^2-2a)$.
21. $2(a+1)^2-2(a+1)(a-1)+3(a-2)^2$.
22. $(a+3)^2-2(a+1)^2+3(a+2)^2$.
23. $(m+n)^3-m(m+n)^2-n(m+n)^2$.
24. $m(m+n)^2+n(m-n)^2-(m+n)^3$.
25. $(a-m)(a-n)-(a-m)(a-p)-(m-a)(n-p)$.
26. $(m+n+p)^2-m(n+p-m)-n(m+p-n)$.
27. $(3m+1)(3m-1)+[mn-\{1-m(2n-9m)\}]$.
28. $3a-[2a+3(a-1)-2(3-2a)]-4$.
29. $2[3a-4(2a-1)-3(-2a-\overline{a-3})]$.
30. $116-5[3-2\{8+3(4-2[8-\overline{3+x}])\}]$.

SIMPLE EQUATIONS

Exercise 12

Solve the following equations:

1. $3x+4=2x+5$.
2. $5x+9=2x+15$.
3. $3x-4=x+12$.
4. $4x+3=x+6$.
5. $5x+7=2x+9$.
6. $2(x+3)=4+(x-2)$.
7. $5(x-2)=3(x+1)-13$.
8. $7x-(x-3)-12=2x$.
9. $3(x+2)+x^2=5+x^2$.
10. $(x+1)(x+2)=x(x+1)$.
11. $4+5(x+2)-9x=(x+2)^2-x^2$.
12. $(x+2)(x-5)=(x+4)(x-1)$.
13. $(x-1)(x+3)-2(x+1)(x-5)+x^2=0$.
14. $2(x^2+2x+1)-(x+2)=2x^2+6$.
15. $(x+4)^2+(x+1)^2=(x+3)^2+2x(x+1)-x^2$.
16. $3(x+5)(x+2)-(x+3)(x-1)=2x^2-(x+7)$.
17. $(4x-1)(x+3)-4x^2-(-10x+3)+6=0$.
18. $2[x+x(x-3)+1]\equiv(2x+5)(x-1)$.
19. $5\{2(x+1)-(x+3)\}=3[x+2\{x-5(3-x)\}]$.
20. $2[3x^2+(x-2)(x-1)]=3[2x^2+(x-3)]+2x^2$.
21. $3[5x-(x+3+\overline{2x-1})]=3x-4\{3x+2(x-\overline{1+3x})\}$.
22. $[(x-2)(x+1)+(x+3)(x+2)]=[(x+3)(x-5)+(x-5)(x+1)]$.
23. $(x+2)(2x+1)(3x+3)=(6x-3)(x+1)(x+3)$.
24. $ax+a=4a$.
25. $2ax+c=5c$.
26. $4ax-5c=5a-5c$.
27. $2cx+d=4c^2+d$.
28. $ax-(a+b)=3a+b$.
29. $2(a-x)=8a$.

30. $3(x+a)+2(x-a)=6(a-x)$.

31. $(a+b)x+(a-b)x=a^2b$.

32. $5a+(a+bx)c=ac-bcx$.

33. $10(a+b)+3x=a+b-5x$.

34. $3(a+b)x-2(a-b)x=a+5b$.

35. $(x-m)(x-n)=(x-m-n)^2$.

PROBLEMS IN SIMPLE EQUATIONS

Exercise 13

1. What number is that which, when doubled, equals 24?

2. What is the number that, increased by 12, equals 27?

3. If a certain number is increased by 12, twice the sum will be 28. What is the number?

4. Four times a certain number when diminished by 6 is equal to 12 more than the number. What is the number?

5. There are two numbers whose sum is 77, and the greater is 13 more than the smaller. Find them.

6. A man is 13 years older than his brother, and the sum of their ages is 49 years. Find the age of each.

7. A father is 4 times as old as his son, and the sum of their ages is 55 years. Find the age of each.

8. The sum of the ages of three brothers is 85 years. The oldest is twice the age of the youngest and 5 years older than the second. Find the age of each.

9. A child is 3 years older than his brother, and 5 times his age is 3 years more than 6 times his brother's age. Find the age of each.

10. Five years ago a man was 4 times as old as his son, but now he is only 3 times as old. Find the present age of each.

11. A man bought the same number each of 2-cent, 5-cent, and 6-cent stamps, paying \$0.91 for the lot. How many of each kind did he buy?

12. Find three consecutive numbers whose sum is 39.

13. Find three consecutive odd numbers whose sum is 33.

14. Find two consecutive even numbers, the difference of whose squares is 52.

15. A man bought a number of horses at \$150 each, twice as many cows at \$40 each, and 3 times as many sheep at \$5 each. The lot cost \$1225. How many of each kind did he buy?

16. How can you pay a bill of \$80 so as to use the same number each of 1-dollar, 5-dollar, and 10-dollar bills?

17. A man asked a farmer how many cows he had, and was answered, "If you gave me 18 more, I should then have twice as many as I now have." How many had he?

18. A man sold 15 hens, receiving 80 cents each for a part and 50 cents each for the remainder. He got \$9.60 for all. How many were sold at each price?

19. Three dollars in nickels and dimes were distributed among 42 boys, and each boy received one coin. How many boys received dimes?

20. Into what two amounts must \$1700 be divided so that the income of one part at 5 per cent interest shall be double the income of the other part at 6 per cent interest?

REVIEW

Exercise 14

1. If $a=3$, $b=1$, $c=0$, and $d=1$, find the value of

$$a-(a-b)+\{a-(b+c)\}-[a-(b-\overline{c-d})].$$

2. From what expression must you subtract the sum of $5a^2+8$, $3a+2a^2$, and a^2+a-1, to produce the expression $4a^2-8a+3$?

3. If $a=m+n-2p$, $b=m-2n+p$, and $c=n+p-2m$, show that $a+b+c=0$.

4. What must be added to a^3+a^2-2a+3 that the sum may be $-a^3-a^2+2a-3$.

5. To what expression must $x^4-3x^3+2x^2-x+5$ be added to produce x^3-x-1?

6. What is the numerical value of the remainder when $3a+2c-d$ is subtracted from $4a+3c-2d$ if $a+c=d$ and $d=7$?

7. If $A=x^3-1+4x^2$, $B=-x-2x^2+1$, and $C=2x^3+2x^2+x+1$, find the value of $-A-[B-(2A-C)+C]$.

8. Simplify

$$4a-[-\overline{6c-(-5+2b-3d)-4a}]-\overline{5-(4c-3d)}.$$

9. Simplify

$$1-[-1+\{-1-(-1+\{-1+(-1)\}-1)\}].$$

10. What is the coefficient of x in the reduced form of the expression $(x-4a)-[2a-3\{x-2(x-a)\}]$?

11. Multiply $m^4 - 2m^3 + 2m^2 - 2m + 1$ by $m^4 + 2m^3 + 3m^2 + 2m + 1$.

12. Multiply $a^4b - a^3b^2 + a^2b^3 - ab^4$ by $a^3b - a^2b^2 + ab^3$.

13. Multiply $\frac{1}{8}a^3 - \frac{2}{3}a^2 + a - 1$ by $\frac{1}{2}a - 1$.

14. Multiply $0.1x^2 + .04x + 0.5$ by $0.1x^2 + 4x + .05$.

15. Divide $6m^4 + m^3 - 29m^2 + 27m - 9$ by $3m^3 + 5m^2 - 7m + 3$.

16. Divide $1.2x^4 - 2.9x^3 + .9x^2 + x$ by $.3x - .5$.

17. Multiply by inspection $(a + b - 2)(a - b - 2)$.

18. Multiply by inspection $(a + b - c)(a - b + c)$.

19. Expand by inspection $(a - 2b + 3c - d)^2$.

20. Divide 1 by $1 - 3m$ to 5 terms.

21. Divide m^4 by $m + 2$ to 4 terms.

22. Simplify $(5a + 1)(a - 3) - (2a - 3)^2 - (a - 5)(a + 3)$.

23. Simplify $(2a - 1)(a + 4) - 2a^2 - \{3a + (2a - 1)(a - 6)\}$.

24. Simplify $5a + (4a - 1) + a + 3(a + 1) - (a + 3)(a + 1)$.

25. Simplify $4a^2 - 3a[a^3 + a^2 - (a - 2)] - 3(a + 1)(a - 7)$.

26. Find the value of $(a + b)^2 - (a + c)^2 - 2(a + b + c)$ when $a = 1$, $b = 2$, and $c = 0$.

27. Find value of $\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$, when $a = 5$, $b = -11$, and $c = 2$.

28. Find value of $\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$, when $a = 4$, $b = -11$, and $c = -3$.

29. $[(x + a)^2 + 5(x + a) + 4] \div [(x + a) + 1] = ?$

30. $[5(x + m) + 3][5(x + m) - 3] = ?$

31. Solve $2a - 3x(a + c) = 5a + 3c$.

32. Solve $x^2-(x+a)^2=(a+1)^2$.

33. Solve $a^2+(x-1)(x-2)=x^2+(a-1)(a+1)$.

34. Solve $4(x+4)(x-3)-2(x^2-2)=3(x+1)(x-4)-x^2$.

FACTORING

Exercise 15

Factor:

1. a^3+a^2+a.
2. m^3-m^2+m.
3. $2m+4m^2+6m^3$.
4. $5c^2+10c+15$.
5. $a^2+4a^3+6a^4$.
6. $3x^2+9x^3-6x^4$.
7. $10x^3-12x^2+13x$.
8. $8x^4-12x^3-16x^2$.
9. $4a^3+8a^2+12a$.
10. $6a^3-9a^2+3a$.
11. $5m-10m^2+15m^3$.
12. $12m^2-18m^4+24m^6$.
13. $5ac-10bc+5cd$.
14. $4a^2c-10bc^2+6ac$.
15. $6x^2y+3xy^2-9x^2y^2$.
16. $m^3-3m^4+4m^5-m^6$.
17. $a^3c-a^2c^2+a^3c^3+ac^4$.
18. $8a^2+4a^3-2a^4+6a^5$.
19. $a^4b-a^3b^2+a^2b^3-ab^4$.
20. $4a^4b-12a^4b^2-16a^4b^3+8a^4b^4$.
21. $15a^2y^4-150a^2y^6-225ay^7+15ay^4$.
22. $48m^2n^6-144m^3n^3-192m^3n^3+240m^{10}n^{11}$.

Exercise 16

Factor:

1. a^2+4a+4.
2. a^2+6a+9.
3. $a^2-8a+16$.
4. $x^2-20x+100$.
5. $a^2-18a+81$.
6. $4m^2+4m+1$.

7. $a^2 + 22\,a + 121$.
8. $16\,c^2 + 8\,c + 1$.
9. $a^2 - 36\,a + 324$.
10. $36\,a^2 - 12\,a + 1$.
11. $9 - 6\,a + a^2$.
12. $m^2 + 42\,m + 441$.
13. $a^2b^2 - 14\,ab + 49$.
14. $x^2y^2 + 32\,xy + 256$.
15. $a^2c^2d^2 - 10\,acd + 25$.
16. $64 - 16\,mn + m^2n^2$.
17. $4\,a^2 + 12\,a + 9$.
18. $25\,a^2 - 30\,a + 9$.
19. $9\,a^2 - 30\,a + 25$.
20. $49\,c^2 - 84\,c + 36$.
21. $16\,a^2b^2 + 8\,abc + c^2$.
22. $a^3 + 18\,a^2 + 81\,a$.
23. $36\,a^2 + 60\,a^2c + 25\,a^2c^2$.
24. $144\,m^2 - 240\,mn + 100\,n^2$.
25. $121\,x^4 - 374\,x^2 + 289$.
26. $625\,m^2 - 50\,m + 1$.
27. $(a+b)^2 + 2\,(a+b) + 1$.
28. $(a+b)^2 + 6(a+b) + 9$.
29. $(a-c)^2 - 6\,(a-c) + 9$.
30. $25\,(a-m)^2 - 70\,(a-m) + 49$.

Exercise 17

Factor:

1. $a^2 - b^2$.
2. $a^2 - 4$.
3. $a^2 - 4\,m^2$.
4. $c^2 - 9\,d^2$.
5. $x^2 - 16$.
6. $m^2 - 49$.
7. $9\,a^2 - 16$.
8. $25\,a^2b^2 - 9$.
9. $36\,c^2 - 25$.
10. $36\,c^2d^2 - 9$.
11. $64\,x^2 - 25$.
12. $a^4 - 1$.
13. $a^4 - 81\,c^2$.
14. $a^4x^4 - 25$.
15. $9\,a^4 - 4\,a^6$.
16. $6\,a^3 - 24\,a$.
17. $3\,x^3 - 75\,x$.
18. $a^4b^4 - 81$.
19. $121\,a^2 - 49$.
20. $64\,a^4 - a^8$.
21. $81\,a^3 - 49\,a$.
22. $81\,m^4n^4 - 16$.
23. $324 - 256\,x^6y^6$.
24. $289 - 16\,m^2$.
25. $(a+b)^2 - 1$.
26. $(m-n)^2 - 4$.
27. $m^2 - (n+p)^2$.
28. $9\,a^2 - (b-c)^2$.
29. $4\,(a+b)^2 - c^2$.
30. $16\,(x-y)^4 - 9$.

31. $4a^2-9(a+1)^2$.
32. $9a^2-16(a+2)^2$.
33. $25(a-b)^2-(c+d)^2$.
34. $36(a+b)^2-49(m-n)^2$.
35. $(a^2-b^2)^2-4(a^2+b^2)^2$
36. $100-(a+b+c)^2$.
37. $81(a+b)^2-4(a+b+1)^2$.
38. $9(a^2+b+c)^2-16(a^2-b-c)^2$.

Exercise 18

Factor:

1. $a^2+2ab+b^2-c^2$.
2. $m^2-2mn+n^2-p^2$.
3. $x^2-b^2+y^2-2xy$.
4. $m^2+n^2-p^2+2mn$.
5. $m^2+n^2-p^2-2mn$.
6. $2m+n^2-m^2-1$.
7. $1-x^2-2xy-y^2$.
8. x^2-4a^2-4a-1.
9. $2mn+m^2n^2+1-p^2$.
10. a^2-c^2-1-2c.
11. $c^2-6cd+9d^2-16m^2$.
12. $4ab-4a^2+1-b^2$.
13. $m^2-4mn-9m^2n^2+4n^2$.
14. $9x^2-25z^2+16y^2+24xy$.
15. $4a^2+12ab-9c^2+9b^2$.
16. $20mn+p^2-4m^2-25n^2$.
17. $4a^2+a^4-4a^3-1$.
18. $5x^2-5-5a^4-10a^2$.
19. $8ac-4a^2-4c^2+4$.
20. $m^2+16n^2-16+8mn$.
21. $a^2+2ab+b^2-m^2-2mn-n^2$.
22. $4a^2-4a+1-9x^2+6xy-y^2$.
23. $9a^2-30a+25-4b^2-4b-1$.
24. $a^2-c^2+b^2-d^2-2ab-2cd$.
25. $a^4m^4-10a^2m^2y-n^2-1-2n+25y^2$.
26. $m^2-n^2-x^2+y^2-2(my-nx)$.
27. $25a^4+1-16a^6-9c^2-10a^2-24a^3c$.

28. $5a^2+5b^2-5m^2-10(ab+mn)-5n^2$.

29. $-12ab+2+24a^2b-18b^2+18a^2b^2-8a^4$.

30. $3a^2+12b^2-12a^4-147b^2-84a^2b-12ab$.

Exercise 19

Factor:

1. $m^4+m^2n^2+n^4$.
2. $x^4-7x^2y^2+y^4$.
3. $x^4-5x^2y^2+4y^4$.
4. m^4-23m^2+1.
5. a^4-79a^2+1.
6. m^4-171m^2+1.
7. $25a^4+66a^2b^2+49b^4$.
8. $49x^4-11x^2y^2+25y^4$.
9. $16x^4-73x^2+36$.
10. $49a^4-74a^2b^2+25b^4$.
11. $289m^4-42m^2n^2+169n^4$.
12. $16a^4-145a^2b^2+9b^4$.

Exercise 20

Factor:

1. a^2+3a+2.
2. a^2-a-12.
3. $a^2-9a+20$.
4. $x^2+5x-24$.
5. $a^2+18a+17$.
6. $c^2-11c+24$.
7. $m^2-19m+88$.
8. $c^2-9c-22$.
9. $x^2+5x-14$.
10. $a^2-3a-28$.
11. $x^2+9x+14$.
12. $y^2-11y+28$.
13. $x^2-9x+14$.
14. $c^2+42c-43$.
15. $m^2-4m-165$.
16. $y^2+12y-108$.
17. $a^2x^2-21ax-46$.
18. $a^2b^2+13ab+40$.
19. $a^2m^2+21am-130$.
20. $c^2d^2+9cd-52$.
21. $m^4n^2-2m^2n-35$.
22. $x^2z^2-20xz-69$.

23. $x^2y^2 - xy - 72$.
24. $x^4 - 13x^2 + 36$.
25. $x^5 - 9x^3 - 22x$.
26. $x^4 - 8x^2 - 9$.
27. $a^3 - 7a^2 - 78a$.
28. $a^2b^2 - 6ab - 187$.
29. $16 - 6a - a^2$.
30. $18 - 19c + c^2$.
31. $147 - 46x^3 - x^6$.
32. $90m^2 + 13m^4 - m^6$.
33. $x^3 + 10x^2 - 96x$.
34. $m^5 - m^4 - 156m^3$.
35. $x^2 + (a+b)x + ab$.
36. $x^2 + (m+n)x + mn$.
37. $a^2 + (c+d)a + cd$.
38. $x^2 - (m+n)x + mn$.
39. $x^2 + (a+2b)x + 2ab$.
40. $x^2 + (3a+2b)x + 6ab$.
41. $x^2 - (a-b)x - ab$.
42. $x^2 - ax - bx + ab$.
43. $a^2 - (3m-2)a - 6m$.
44. $a^2 + (m - 2mx^2)ay - 2m^2x^2y^2$.

Exercise 21

Factor:

1. $2a^2 + 5a + 3$.
2. $6x^2 - x - 2$.
3. $2x^2 - 3x - 9$.
4. $2a^2 + 7a + 3$.
5. $8x^2 + 10x - 3$.
6. $6m^2 + 13m - 5$.
7. $15x^2 + 11x + 2$.
8. $7x^2 - 41x - 6$.
9. $6a^2 - 29a + 28$.
10. $3a^2 - 19a + 6$.
11. $12c^2 + 17c - 5$.
12. $6y^2 - y - 12$.
13. $3m^2 - 11m + 6$.
14. $8a^2 - 30a - 8$.
15. $24m^2 - 14m - 49$.
16. $2x^2 + 7x - 15$.
17. $18a^2 + 9a - 2$.
18. $40x^2 - 61x + 7$.
19. $8m^2 + 2m - 3$.
20. $35a^2 - 13a - 12$.
21. $6a^2 + 25a - 9$.
22. $8m^2 + 5m - 3$.
23. $42x^2 - 11x - 20$.
24. $16m^2 - 6m - 27$.
25. $12y^2 - y - 20$.
26. $2x^2 - 4x - 126$.

27. $12m^3 - 7m^2n - 12mn^2$.
28. $6m^3 + 29m^2 - 22m$.
29. $8x^4 - 26x^2 + 18$.
30. $a^2m^3 - 9a^2m^2 + 20a^2m$.
31. $26a^3 + 197a^2 + 15a$.
32. $52a^2 - 153a - 52$.
33. $3m^2 - 30m + 63$.
34. $12x^4 - 25x^2 + 12$.
35. $x^5 - x^4 - 42x^3$.
36. $20a^3b^3 - 9a^2b^2 - 20ab$.
37. $16c^2d^2 + 2cd - 3$.
38. $75a - 210ax + 147ax^2$.
39. $48x^5 - 176x^4 + 65x^3$.
40. $55x^5 - x^4 - 2x^3$.
41. $2(a+1)^2 + 3(a+1) + 1$.
42. $3(a+1)^2 - 8(a+1) + 4$.
43. $2(a+1)^2 + 5(a+1) + 2$.
44. $2(a-2)^2 - 5(a-2) - 3$.
45. $3(m-1)^2 - 11(m-1)n + 6n^2$.
46. $10(a+b)^2 - 11(a^2 - b^2) + 3(a-b)^2$.

Exercise 22

Factor:

1. $m^3 - n^3$.
2. $m^3 - 27$.
3. $8a^3 + 1$.
4. $27 - 8c^3$.
5. $27a^3 + 8$.
6. $64 - 125c^3$.
7. $125a^3 + 27b^3$.
8. $a^3b^3 - 8$.
9. $27 - 8c^3d^3$.
10. $m^3n^3 - 343$.
11. $64a^3 + 125$.
12. $8a^6 - 27$.
13. $64a^9 + c^6$.
14. $125m^{12} - n^{15}$.
15. $343a^3m^3 - 729n^6$.
16. $(m+n)^3 + x^3$.
17. $(c+d)^3 - 8$.
18. $(a+1)^3 + 64$.
19. $27 - 8(a+2)^3$.
20. $64(a+b)^3 - 27a^3$.
21. $(a+1)^3 - 8(a+2)^3$.

---

22. $a^4 - b^4$.
23. $a^4 - 16$.
24. $81 - m^4$.
25. $a^6 - 64$.
26. $a^{10} - 25$.
27. $4m^4 - 81$.
28. $a^{10} - m^{10}$.
29. $m^{12} - n^{12}$.
30. $(a-b)^4 - 1$.
31. $(a-b)^6 - 8$.
32. $(a+b)^4 - 256$.
33. $(2a+1)^4 - 16(2a-1)^4$.

34. m^3+8.
35. m^5+1.
36. x^5-1.
37. x^5+32.
38. $x^{10}+y^{10}$.
39. x^6-y^6.
40. x^6+y^6.
41. x^8-y^8.
42. $x^{12}+1$.
43. m^6+27.
44. $64\,a^6+1$.
45. $a^{15}+m^{15}$.

Exercise 23

Factor:

1. $m^2+mn+mp+np$.
2. $ab+a+7\,b+7$.
3. $2\,cm-3\,dm+2\,c-3\,d$.
4. $ay-ab-bx+xy$.
5. x^3+x^2+x+1.
6. $xy-2\,y-x^2+2\,x$.
7. a^3-a^2+a-1.
8. m^2-n^2-m-n.
9. a^3-a^2-a+1.
10. $6\,a^3+4\,a^2-9\,a-6$.
11. m^3+m^2-m-1.
12. $15\,ax-20\,ay+9\,bx-12\,by$.
13. $m^3+5\,m^2+2\,m+10$.
14. $m^2n-p^2mn+mx-p^2x$.
15. $a^3-1+2\,(a^2-1)$.
16. $a^2\,(a^2-9)-a\,(a+3)^2$.
17. $5\,(x^3+8)-15\,(x+2)$.
18. $x^4+2\,x^3-8\,x-16$.
19. $2\,a^2\,(a+3)-3\,a^2-8\,a+3$.
20. $c^3+4\,c^2-3$.
21. $m^3-19\,m-30$.
22. $4\,a^3-39\,a+45$.
23. $a^3+9\,a^2+11\,a-21$.
24. $a^3+3\,a^2+3\,a+2$.
25. $am+an+ap+bm+bn+bp$.
26. $m^2\,(n-x)+m\,(n-x)-2\,(n-x)$.
27. $m^2+y^2+m-2\,my-y-6$.
28. $(a-1)\,(a+1)+(x-1)\,(a-1)$.
29. $(a+1)\,(x+2)-(a+1)\,(y+2)$.
30. $a^2-1-x\,(a-1)$.
31. $m^3+1-b\,(m+1)$.
32. $(m+1)\,(m^2-4)-(m+1)\,(m+2)-m-2$.

REVIEW

Exercise 24

Factor:

1. $6a^2+19a+10$.
2. $2x^2-6xy-140y^2$.
3. a^4+a.
4. $c^4+c^2d^2+d^4$.
5. m^3-m^2-30m.
6. c^4+cd^3.
7. $a^2+2xy-x^2-y^2$.
8. $x^4-x^3a+xz^3-az^3$.
9. $x^3-x^2y+xy^2-y^3$.
10. $1-m^2-2mn-n^2$.
11. $c^4+d^4-18c^2d^2$.
12. $m^2n^2p^2-m^2p-n^2p+1$.
13. $6c^2-c(d-1)-(d-1)^2$.
14. a^2-c^2-4c-4.
15. $d^2+3d^3-d^4-3d$.
16. $8x^5+27x^2$.
17. x^6+y^3.
18. $24a^2b^2-36b^4-30ab^3$.
19. $7x^7-7x$.
20. $m^5-m^4-2m^3$.
21. $m^4+n^4-23m^2n^2$.
22. $4y+x^2-1-4y^2$.
23. $(2m-5n)^2-(m-2n)^2$.
24. $4a^4n^2-4a^2n^4+8a^3n^3-8an^5$.
25. $25a^2b^2c^2-9$.
26. a^4-8a-a^3+8.
27. x^6-y^6.
28. $m^3-m^2n+mn^2-n^3$.
29. $81m^5-16mn^4$.
30. $72a^3+5a^2-12a$.
31. $1+ax-(c^2+ac)x^2$.
32. $54-16m^3n^3$.
33. $4x^2y^2-(x^2+y^2-z^2)^2$.
34. $49x^4+34x^2y^2+25y^4$.
35. $(x-m)(y-n)-(x-n)(y-m)$.
36. $15m^7-14m^5-8m^3$.

37. $(m-n)(2a^2-2ab)+(n-m)(2ab-2b^2)$.

38. $(a-1)(a-2)(a-3)-(a-1)+(a-1)(a-2)$.

39. $(2c^3+3d^2)a+(2a^2+3c^2)d$.

40. d^4+d^3-d-1.

41. x^6-64.

42. $1000+27c^6$.

43. $5a^3-20a^2-300a$.

44. $mn-pr+pn-mr$.

45. $c^2-2cd+d^2-1$.

46. $4-9(x-3y)^2$.

47. $a^2x^2-a^2-x^2-2ax$.

48. $18y^3-48y^2-18y$.

49. $m^4-2mn^3-n^4+2m^3n$.

50. $x^4+125xy^6$.

51. $m^3+m^2n+2mn^2+2n^3$.

52. $24x^2-5x-36$.

53. $72x^2+41x-45$.

54. $(a^2-b^2)^2-(a^2-ab)^2$.

55. $ac^2+7ac-30a$.

56. $x^2y^2-x^2-y^2+1$.

57. $x^4+4x^3-8x-32$.

58. $5c^2x^2+35cx^2-90x^2$.

59. $dc+cd-ab-bd$.

60. x^2-m^2+x-m.

61. $9x^4-66x^2+25$.

62. $2m^3n-16n^{16}$.

63. $64x^7-x$.

64. $(a^2+a-1)^2-(a^3-a-1)^2$.

65. x^4-27x^2+1.

66. $81x^4-y^8z^{12}$.

67. $5cd-12d^2+2c^2$.

68. $72x^3+66x^2-40x$.

69. $3x^4+192xy^6$.

70. $3c^3-12c^2d^2-4d^2+1$.

71. $p^3q^3-27r^3$.

72. $m^2x^3-x^3-m^2+1$.

73. $x^4-25(x-3)^2$.

74. a^9-c^6.

75. $5m^3-5m^2n-5mn-5m$.

76. m^4-27m^2+162.

77. $9x^4+68x^2-32$.

78. $1-m^2n^2-p^2r^2+2mnpr$.

79. m^4-5m^2+4.

80. m^3-m^2x-m+x.

81. $24x^3+43x^2-56x$.

82. $m^4-(m-6)^2$.

83. $8a^{13}+am^{12}$.

84. $24\,c^2d^2 - 47\,cd - 75.$

85. $16\,x^2 - y^2 - 9 + 6\,y.$

86. $12\,a^3 + 69\,a^2 + 45\,a.$

87. $a^2c + 3\,ac^2 - 3\,a^3 - c^3.$

88. $c^9 - 64\,c^3 + 64\,a^3 - a^9.$

89. $a^4 - a^2b^2 - b^2 - 1.$

90. $2\,x^3 + 3\,x^2 - 8\,x + 3.$

91. $10\,(m+c)^2 + 7\,a\,(m+c) - 6\,a^2.$

92. $100 + 10\,x^4 - 25\,x^6 - x^2.$

93. $(m+p)^2 - 1 - 2\,(m+p+1).$

94. $a^2 + 2\,a - c^2 + 4\,c - 3.$

95. $(2\,m-3)^2 - 6\,(2\,m-3)\,n - 7\,n^2.$

96. $x^6 - 13\,x^3 + 12.$

97. $m^2 + n^2 - (1 + 2\,mn).$

98. $(c-2\,d)^2 - 9 - 3\,(c-2\,d+3).$

99. $6\,c^3 - 25\,c^2 + 8\,c - 16.$

100. $x^2y + y^2z + xz^2 - x^2z - xy^2 - yz^2.$

HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE

Exercise 25

By factoring find the H. C. F. and L. C. M. of:

1. $a^3 - b^3,\ a^2 - b^2,\ a^4 - b^4.$

2. $a^3 - ab^2,\ 2\,a^4 - 2\,a^2b^2,\ a^3 - 2\,a^2b + ab^2.$

3. $a^4 - 16,\ a^2 - a - 2,\ a^2 - 4\,a + 4.$

4. $m^2 - 3\,m + 2,\ m^2 - m - 2,\ m^2 + m - 6.$

5. $x^2 + x - 12,\ x^2 - 4\,x + 3,\ x^2 + 2\,x - 15.$

6. $a^3+2a^2-15a,\ a^3-5a^2+6a,\ a^3-2a^2-3a.$
7. $x^4-3x^2+1,\ 2x^3+2x^2-2x,\ x^3+2x^2-1.$
8. $x^4+x^2-6,\ 3x^4+6x^2-24,\ x^4-10x^2+16.$
9. $2x^2-11x-40,\ 3x^2-25x+8,\ x^2-x-56.$
10. $5m^3-5n^3,\ 15(m-n)^3,\ 10m^2-20mn+10n^2.$
11. $m^4-n^4,\ m^3+m^2n-mn^2-n^3,\ m^4-2m^2n^2+n^4.$
12. $5a^3+40b^3,\ 7a^3+28a^2b+28ab^2,\ 3a^4-12a^2b^2.$
13. $c^4-d^4,\ c^5+d^5,\ c^3+d^3,\ c^2+2cd+d^2.$
14. $12x^2-30x-18,\ 27x^2-90x+27,\ 15x^2-42x-9.$
15. $mn-mp+2n-2p,\ m^3+6m^2+12m+8.$
16. $x^4+x^2y^2+y^4,\ x^2z+x^2v+xyz+xyv+y^2z+y^2v.$
17. $12(m^6-n^6),\ 18(m^4-n^4),\ 24(m^2-mn+n^2).$
18. $a^3-b^2-a^2b+ab,\ a^3+b^3-a^2b-ab^2,\ a^4-ab^3+b^4-a^3b.$
19. $a^3-3a^2-4a+12,\ a^4-13a^2+36,\ a^3+2a^2-9a-18.$
20. $a^3+3a^2+3a+2,\ a^3-8a-8,\ a^3+3a^2+a-2.$
21. $a^3-22a+15,\ a^3+6a^2-25,\ a^3+13a^2+36a-20.$

Exercise 26

Find the H. C. F. and the L. C. M. of:

1. $x^3+4x^2+7x+6,\ x^3+4x^2+x-6.$
2. $x^3+6x^2+11x+12,\ x^3+2x^2-6x+8.$
3. $2x^4-x^3+x^2-x-1,\ 2x^4+3x^3-x^2+x+1.$
4. $3x^4+3x^3-3x-3,\ 4x^4-4x^3-8x^2+4x+4.$
5. $6x^3+19x^2+19x+6,\ 4x^3+8x^2+5x+3.$
6. $x^4+3x^3+5x^2+4x+2,\ x^4+3x^3+6x^2+5x+3.$
7. $2m^4+m^3-9m^2+8m-2,\ 2m^4-7m^3+11m^2-8m+2.$

8. $2a^4+5a^3+2a^2-a-2$, $6a^5+3a^4+6a^3-3$.

9. $3a^3+14a^2-5a-56$, $6a^3+10a^2+17a+88$.

10. $4m^4+3m^3-6m^2-29m+30$, $4m^4-m^3-13m^2+14m-5$.

11. $4a^5+14a^4+20a^3+70a^2$, $6a^6+21a^5-12a^4-42a^3$.

12. $a^4-4a^3b+6a^2b^2-ab^3-6b^4$, $a^3-3a^2b+3ab^2-2b^3$.

13. $c^5-2c^3-7c^2+8c-10$, $c^4+c^3-9c^2+10c-8$.

14. $2m^4-m^3n-11m^2n^2+17mn^3-7n^4$, $m^4-2m^3n-m^2n^2+4mn^3-2n^4$.

15. $x^3+2x^2-10x-21$, $x^3+4x^2-2x-15$, $x^3+2x^2-7x-12$.

16. $2a^2(a^4-2a^3-7a^2+16a+7)$, $5a^3(a^3-5a^2-23a-8)$, $6a(a^3-6a^2-26a-9)$.

FRACTIONS

I. TRANSFORMATIONS

Exercise 27

Reduce to lowest terms:

1. $\dfrac{x^3+27}{x^2-9}$.

2. $\dfrac{4x^2+5x+1}{5x^3+5}$.

3. $\dfrac{12x^2-x-6}{12x^2-13x+3}$.

4. $\dfrac{3m-6}{6m^2-24}$.

5. $\dfrac{x^4-81}{x^4+18x^2+81}$.

6. $\dfrac{16-(m+n)^2}{(m-4)^2-n^2}$.

7. $\dfrac{2a^2-2a-12}{a^3+2a^2-a-2}$.

8. $\dfrac{x^2+mx+nx+mn}{x^2+3mx+2m^2}$.

9. $\dfrac{4m^2-8mn+4n^2}{4m^3-4m^2n-4mn^2+4n^3}$.

10. $\dfrac{a^4+a^3+a^2+a+1}{a^5-1}$.

11. $\dfrac{m^6-64}{m^4+2m^3-8m-16}$.

12. $\dfrac{m^8-m^6n^2+m^2n-n^3}{m^4-n^4}$.

13. $\dfrac{m^2+n^2+p^2+2\,mn+2\,mp+2\,np}{m^2-n^2-p^2-2\,np}$.

14. $\dfrac{6\,ac-2\,ad-3\,bc+bd}{9\,ac-3\,ad+3\,bc-bd}$.

15. $\dfrac{(m+n)^2+7\,m+7\,n+10}{m^2+2\,mn+n^2-4(m+n)-12}$.

16. $\dfrac{[(x+y)^2-z^2]\,xyz}{(x^2-y^2-z^2)^2-4\,y^2z^2}$.

17. $\dfrac{10\,x^5+5\,x^4-105\,x^3}{24\,x^4-64\,x^3-24\,x^2}$.

18. $\dfrac{m^4-m^3-3\,m+3}{2\,m^3-m-1}$.

19. $\dfrac{x^3-11\,x+6}{x^3-20\,x+33}$.

20. $\dfrac{2-5\,a-4\,a^2+3\,a^3}{4+4\,a+9\,a^2+4\,a^3-5\,a^4}$.

21. $\dfrac{x^3+7\,x^2+12\,x+4}{x^3+8}$.

22. $\dfrac{m^3-8\,m-3}{m^4-7\,m^2+1}$.

23. $\dfrac{x^3+2\,x^2-7\,x-2}{x^4-14\,x^2+1}$.

24. $\dfrac{5\,m^5-5\,m^3n^2}{2\,m^3-2\,m^2n+2\,mn^2}$.

Exercise 28

Change to mixed expressions:

1. $\dfrac{x^2+5\,x+10}{x+2}$.

2. $\dfrac{x^2-12\,x-47}{x+3}$.

3. $\dfrac{x^3-2\,x^2+4\,x-1}{x-3}$.

4. $\dfrac{x^4-2\,x^3+x^2-x+7}{x^2+x-3}$.

5. $\dfrac{4\,x^5+12\,x^3-x^2+4}{2\,x^3+7}$.

6. $\dfrac{x^4-x^3+3\,x+2}{x^2+3}$.

7. $\dfrac{3\,a^3}{a-b}$.

8. $\dfrac{m^4+2}{m^2+m-1}$.

9. $\dfrac{x^3}{x^2+x-1}$.

10. $\dfrac{12}{2+x-x^2}$.

11. $\dfrac{x^5}{x^2+2\,x+2}$.

12. $\dfrac{1}{1-x}$ to 4 terms.

Exercise 29

Change to improper fractions:

1. $x+1-\frac{x-1}{x+1}$.

2. $a-3c+\frac{2ac+4c^2}{a+c}$.

3. $1-\frac{(a+b)^2}{a^2+b^2}$.

4. $1-\frac{a^2+b^2}{(a+b)^2}$.

5. $m^2-n^2+\frac{m^2n^2(m+n)}{m^3-n^3}$.

6. $(c+d)^2-\frac{(c-d)^3}{c+d}$.

7. $a^2-a+1-\frac{a^3-1}{a+1}$.

8. $\frac{x^2(x+3)}{x^2+5x+6}-x+2$.

9. $m^2-2mn+\frac{2mn(m^2-n^2)+m^2n^2}{m^2-n^2}$.

10. $m+n-p-\frac{2mn+n^2-p^2}{m+n+p}$.

11. $\frac{(x^2+3x+1)^2}{x^2+3x-2}-x^2-3x+1$.

12. $a+2-\frac{a^2+6a}{a+4}+a^2$.

13. $1+\left(x^2+x+\frac{1}{x-1}\right)$.

14. $x^2-\left[x-\left(\frac{x-1}{x^2}\right)\right]$.

15. $\left(\frac{x-1}{x+1}-x^2\right)+x^2+2$.

16. $1-\left[x-\left\{2x^2+\frac{(x^2-1)^2-5}{x+2}-x^3\right\}\right]$.

17. $3m-\frac{[(m+2)(m-1)-(-m+1)]}{m+3}-(m+2)$.

18. $\frac{a^2+[-ab-a^2+(-a^2+ab)-b^2]}{a-b}+a-b$.

19. $(a-b)^2-\left[(a+b)+\frac{3ab(b-a)+a(a^2-a)-b^3}{a-b}\right]$.

II. ADDITION AND SUBTRACTION

Exercise 30

Collect:

1. $\frac{5x}{3}+\frac{3x}{4}+\frac{x}{2}$.

2. $\frac{4x+1}{4}+\frac{2x-1}{3}$.

3. $\frac{2x+1}{3x^2}+\frac{x-1}{2x^3}$.

4. $\frac{1}{x(x+1)}+\frac{1}{x(x-1)}$.

5. $\frac{2}{x-1}+\frac{1}{x+1}-\frac{1}{x^2-1}$.

6. $\frac{a+1}{a-1}+\frac{a-1}{a+1}$.

7. $\frac{a+2}{a-2}-\frac{a-2}{a+2}$.

8. $\frac{5a+1}{a+3}-\frac{2a+1}{a-3}$.

9. $\frac{a+c}{(a-c)^2}+\frac{c}{a^2-c^2}$.

10. $\frac{(m+n)^2}{m-n}-\frac{(m-n)^2}{m+n}$.

11. $\frac{2}{m+1}+\frac{2}{m-1}+\frac{2}{m-2}$.

12. $\frac{2}{a}+\frac{3}{a+1}+\frac{1}{a-1}$.

13. $\frac{a}{a-1}-\frac{a}{a+1}+\frac{1}{a^2-1}$.

14. $\frac{3}{m}-\frac{5}{m-1}-\frac{2m-3}{m^2-1}$.

15. $\frac{2}{3-3a^2}-\frac{3}{5-5a}+\frac{1}{1+a}$.

16. $\frac{m-n}{mn}+\frac{n-p}{np}+\frac{p-m}{mp}$.

17. $\frac{4a-b}{a+2b}+\frac{18ab}{a^2-4b^2}+\frac{4a+b}{a-2b}$.

18. $\frac{x^2+x}{x^3-1}-\frac{1}{x-1}$.

19. $\frac{c^2+d^2}{cd}-\frac{c^2}{cd+d^2}-\frac{d^2}{cd+c^2}$.

20. $\frac{x^2}{x^2+x+1}+\frac{x}{x-1}+\frac{1}{x^3-1}$.

21. $\frac{1}{x^2+2x+4}-\frac{1}{x-2}+\frac{6}{x^3-8}$.

22. $\frac{2-2m}{(m-1)^3}+\frac{m+1}{(m-1)^2}-\frac{1}{m-1}$.

23. $\dfrac{a+m}{a^2+am+m^2}-\dfrac{a-m}{a^2-am+m^2}$.

24. $\dfrac{x^2-1}{x^3+x}+\dfrac{1}{x}+\dfrac{2x}{x^2+1}$.

25. $\dfrac{x}{x+1}+2-\dfrac{1}{x-1}$.

26. $\dfrac{a}{a+2}+1-\dfrac{a}{a-2}-a$.

27. $\left(3m+\dfrac{2}{n}\right)-\left(2m-\dfrac{1}{n}\right)$.

28. $3c-\dfrac{c}{c+1}+\dfrac{1}{c-1}$.

29. $\left(2a-\dfrac{a+3}{2}\right)+\left(a-\dfrac{a-4}{3}\right)-2a$.

30. $\left(m+\dfrac{n^2}{m-n}\right)-\left(m+\dfrac{n^2}{m+n}\right)$.

31. $\dfrac{x+1}{x+2}-2+\dfrac{x-3}{x-4}$.

32. $2-\left(\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}\right)+\dfrac{4y^2}{x^2-y^2}$.

33. $\dfrac{(m^2+n^2)^2}{mn(m-n)^2}-\left(\dfrac{m}{n}+\dfrac{n}{m}+2\right)$.

34. $\dfrac{x-3a}{x^2-3ax+9a^2}+\dfrac{9ax}{x^3+27a^3}+\dfrac{1}{x+3a}$.

35. $\dfrac{2a}{a^2-1}+\dfrac{3}{1-a}+\dfrac{1}{a+1}$.

36. $\dfrac{5}{2-a}+\dfrac{3}{a+2}-\dfrac{1}{a^2-4}$.

37. $\dfrac{a+1}{a+3}+\dfrac{4}{9-a^2}-\dfrac{a-1}{3-a}$.

38. $\dfrac{1}{x}+\dfrac{3}{x-1}+\dfrac{2}{1-x^2}-\dfrac{2}{x+1}-\dfrac{3}{x+x^2}$.

39. $\dfrac{5}{3m+15}-\dfrac{2}{125-5m^2}+\dfrac{8}{7m-35}$.

40. $\dfrac{m-5}{5+m}+\dfrac{m+5}{5-m}+\dfrac{21m}{m^2-25}$.

41. $\dfrac{1}{a}+\dfrac{20a}{1-16a^2}+\dfrac{2}{8a-2}-\dfrac{1}{a+4a^2}$.

42. $\frac{1}{a+3}+6-\frac{2}{3-a}+\frac{1}{9-a^2}$.

43. $3x+\frac{2}{x-3}-\frac{3x^2}{x+3}-\frac{4}{9-x^2}$.

44. $\frac{m+1}{m^2+m+1}-\frac{1}{m-1}-\frac{m^2+m-2}{1-m^3}$.

45. $\frac{c-x}{c+d}+\frac{c+d}{c-x}-\frac{(d-c)^2}{(c-x)(c-d)}$.

46. $\frac{x-3a}{x^2-3ax+9a^2}+\frac{3ax-2x^2}{x^3+27a^3}+\frac{1}{3a+x}$.

47. $\frac{a+1}{(a-4)(2-c)}+\frac{a+4}{(a-1)(2-c)}$.

48. $\frac{m}{(m-n)(m-p)}+\frac{n}{(n-p)(n-m)}+\frac{p}{(p-m)(p-n)}$.

49. $\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-z)(y-x)}+\frac{z^2}{(z-x)(z-y)}$.

50. $\frac{m-1}{m^2-m+1}+\frac{2}{m^4+m^2+1}+\frac{m+1}{m^2+m+1}$.

51. $\frac{1}{x+1}-\left[\frac{1}{x-1}-\frac{1}{x^2-1}\right]$.

52. $\frac{2}{x+5}-\left\{3-\frac{1}{x-5}-\left(\frac{1}{x^2-25}+\frac{x}{x-5}\right)\right\}$.

53. $\left[2x+2-\left(\frac{3}{x+1}-\frac{1}{x-1}\right)\right]-\frac{2x^3}{x^2-1}-\left\{\frac{1}{x}-\frac{3}{x-1}-\left(\frac{1}{x}-2\right)\right\}$.

54. $\frac{1}{1-x}-\left[\frac{1}{x-2}-\left\{\frac{2}{6-x-x^2}-\left(\frac{1}{2-3x+x^2}+\frac{1}{x-1}\right)\right\}\right]$.

III. MULTIPLICATION AND DIVISION

Exercise 31

Simplify:

1. $\dfrac{m^2-n^2}{m^2}\cdot\dfrac{m^3}{(m-n)^2}$.

2. $\dfrac{a^2-144}{a^2-9}\cdot\dfrac{a+3}{a+12}$.

3. $\dfrac{m^3-n^3}{m^3+n^3}\cdot\dfrac{m^2-mn+n^2}{m-n}\cdot\dfrac{m+n}{m^2+mn+n^2}$.

4. $\dfrac{(c+d)^3}{c^3+d^3}\cdot\dfrac{c^2-cd+d^2}{c^2+2cd+d^2}$.

5. $\dfrac{m^2-4}{(m+2)(m-1)}\cdot\dfrac{m^2-2m+1}{(m+3)(m-2)}$.

6. $\dfrac{mn}{m+n}\cdot\left(\dfrac{m}{n}-\dfrac{n}{m}\right)$.

7. $\left(3a+4-\dfrac{4}{a}\right)\left(3a-4-\dfrac{4}{a}\right)\left(\dfrac{a}{9a^2-4}\right)$.

8. $\dfrac{mn}{m+n}\left(\dfrac{m+n}{n}+\dfrac{n}{m+n}\right)$.

9. $\dfrac{c^4-d^4}{c-d}\cdot\dfrac{c^3-d^3}{c^2-d^2}\cdot\dfrac{c^3+d^3}{c^2+d^2}$.

10. $\left(x^3y^6-\dfrac{1}{x^3}\right)\left(\dfrac{x^4}{x^2y^2(x^2y^2+1)+1}\right)$.

11. $\dfrac{a^2+(m+n)a+mn}{a^2+(m+p)a+mp}\cdot\dfrac{a^2-p^2}{a^2-n^2}$.

12. $\left(9-3m+m^2\right)\left(1+\dfrac{3}{m}+\dfrac{9}{m^2}\right)\left[\dfrac{1}{81+m^2(9+m^2)}\right]$.

13. $\dfrac{3(m-n)}{m(m^2+5m)} \cdot \dfrac{m^2-(p-5)m-5p}{m^2+mp-mn-np} \cdot \dfrac{m+p}{m-p}$.

14. $\dfrac{a^2+ac+ab+bc}{a^2+ac+ad+dc} \cdot \dfrac{a^2+af+ad+df}{a^2+ad+ab+bd}$.

15. $\left(\dfrac{m+n}{m-n}+\dfrac{m-n}{m+n}\right) \div \left(\dfrac{m+n}{m-n}-\dfrac{m-n}{m+n}\right)$.

16. $\left(\dfrac{c+d}{c-d}-\dfrac{c-d}{c+d}\right) \div \left(\dfrac{6}{c}+\dfrac{3}{c-d}-\dfrac{3}{c+d}\right)$.

17. $\dfrac{m^2-9}{m^2-16} \cdot \dfrac{(m-4)^2}{m} \div \left(\dfrac{m^2-7m+12}{m^2} \cdot \dfrac{m+4}{m+3}\right)$.

18. $\dfrac{a^2-a-6}{a^2-a-2} \cdot \dfrac{a^2-2a-3}{a^2+a-2} \div \dfrac{a^2-9}{a^2-2a+1}$.

19. $\left(\dfrac{c+d}{c-d}-\dfrac{c-d}{c+d}\right) \div \left(\dfrac{c^2+d^2}{c^2-d^2}-\dfrac{c^2-d^2}{c^2+d^2}\right)$.

20. $\dfrac{4m^2-n^2-p^2-2np}{4m^2+n^2-p^2+4mn} \div \dfrac{2m-n-p}{2m+n+p}$.

21. $\dfrac{x^2-5x+6}{x^2+3x+2} \cdot \dfrac{x^2+x-2}{x^2+x-6} \div \dfrac{x^2-4x+3}{x^2+4x+3}$.

22. $\left(a+\dfrac{m^2}{a}\right)\left(\dfrac{a^2}{m^2}-1\right) \div \left(1-\dfrac{m^4}{a^4}\right)$.

23. $\dfrac{3cd(d-c)+c^3-d^3}{c^2+d^2+2cd} \cdot \left(c-\dfrac{d^2}{c}\right) \div \dfrac{c^3(c-d)^3}{c^4-d^4}$.

24. $\dfrac{(a-c)^2-b^2}{(b+c)^2-a^2} \cdot \dfrac{(a+b+c)^2}{(a+b)^2-c^2} \cdot \dfrac{(a+c)^2-b^2}{a^2-(b+c)^2}$.

25. $\dfrac{m^4-2m^2n^2+n^4}{m^2+n^2} \cdot \dfrac{m^3-n^3}{m^4-m^2n^2+n^4} \div \dfrac{m^6-n^6}{(m^6+n^6)(m^3+n^3)}$.

26. $\left[\left(1-\dfrac{1}{x}+\dfrac{1}{x^2}\right) \div (1+x+x^2)\right] \div \dfrac{x(x-1)+1}{x^2}$.

27. $\left(\frac{x}{yz}-\frac{y}{xz}-\frac{z}{xy}-\frac{2}{x}\right)\left(1-\frac{2z}{x+y+z}\right)$.

28. $\left(\frac{m}{n}-\frac{4n}{m}\right)^2 \div \left[\left(\frac{m}{n}+\frac{4n}{m}-4\right)\left(\frac{m}{n}+\frac{4n}{m}+4\right)\right]$.

29. $\frac{(c+d)^2-4}{4-(c-d)^2} \cdot \frac{d^2-(c-2)^2}{c^2-(d+2)^2} \div \frac{c+d-2}{c-d-2}$.

30. $\left[\left(\frac{x}{y}-8+\frac{16y}{x}\right)\div\left(\frac{x}{y}-1-\frac{12y}{x}\right)\right]\div\left[\left(1-\frac{4y}{x}\right)\left(\frac{1}{x+3y}\right)\right]$.

31. $\left[\left(\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}\right)\div\frac{x^2-(y+z)^2}{xy}\right]\cdot(x-y-z)$.

32. $\left(\frac{c^2-d^2}{c^3+d^3}\div\left[\left(\frac{1}{d}-\frac{1}{c}\right)\div\left(\frac{c^2+d^2}{d}-c\right)\right]\right)\cdot\frac{1}{c+d}$.

IV. COMPLEX FRACTIONS

Exercise 32

Simplify:

1. $\dfrac{\frac{1}{x}-x}{\frac{1}{x}+1}$.

2. $\dfrac{m+2-\frac{3}{m}}{m-3+\frac{2}{m}}$.

3. $\dfrac{\frac{x+1}{x}-\frac{4}{x+1}}{\frac{1}{x+1}-\frac{1}{x}}$.

4. $\dfrac{\frac{a+2}{a}+\frac{6}{a-3}}{\frac{a^2-a+2}{a-3}+1}$.

5. $\dfrac{x+1-\frac{4}{x-2}}{x+4-\frac{7}{x-2}}$.

6. $\dfrac{x+\frac{6}{x+2}-5}{x-\frac{5}{x+1}-3}$.

7. $\dfrac{\frac{16}{x}-x}{\frac{8}{x^4}+\frac{10}{x^3}-\frac{3}{x^2}}$.

8. $\dfrac{\frac{c}{d^2}-\frac{d}{c^2}}{\frac{1}{d^2}+\frac{1}{cd}+\frac{1}{c^2}}$.

9. $\dfrac{\frac{1}{m}+\frac{1}{n}+\frac{1}{p}}{\frac{m}{n}+\frac{n}{p}-\frac{p}{m}}$.

10. $\dfrac{\dfrac{x^2-5x-14}{x^2-x-6}}{\dfrac{x^2-6x-7}{x^2+6x+5}}$.

11. $3m-\left(\dfrac{2m}{1+\dfrac{1}{2m}}+1-\dfrac{1}{2m+1}\right)$.

12. $\dfrac{\dfrac{c-d}{c+d}+\dfrac{c+d}{c-d}}{\dfrac{c^2-d^2}{(c-d)^2}-1}$.

13. $\dfrac{\dfrac{2}{x-4}-\dfrac{3}{x+1}}{\dfrac{x-14}{\frac{x^2}{3}-x-\frac{4}{3}}}$.

14. $\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{b}{a}+\dfrac{a}{b}}-\dfrac{\dfrac{1}{a}-\dfrac{1}{b}}{\dfrac{b}{a}-\dfrac{a}{b}}$.

15. $\dfrac{\dfrac{a^2+a-2}{a^2-2a-3}}{\dfrac{a^2+3a+2}{a^2-4a+3}}-1$.

16. $1-\dfrac{\dfrac{a^2}{10}+\dfrac{3a}{10}-1}{\dfrac{a^2}{5}+\dfrac{4a}{5}-1}$.

17. $\dfrac{\dfrac{a-c}{a+c}+\dfrac{a^2+c^2}{a^2-c^2}}{\dfrac{a^2}{a-c}+\dfrac{a^2c+c^3}{(a-c)^2}}$.

18. $\dfrac{2}{1-\dfrac{2}{1+\dfrac{2}{1-x}}}$.

19. $\dfrac{\left(\dfrac{x^2}{4}-1\right)-\left(\dfrac{x^2}{9}-1\right)}{\dfrac{5x^2}{36}-1}-1$.

20. $a+\dfrac{1}{a+\dfrac{1}{a+\dfrac{1}{a}}}$.

21. $\dfrac{\dfrac{3(a+2)-2(a+1)}{a^2}}{\dfrac{a^3+64}{a^4-4a^3+16a^2}}$.

22. $\dfrac{m+1}{m+1-\dfrac{1}{m+1+\dfrac{1}{m+1}}}$.

23. $\dfrac{\dfrac{(3x-5)^2}{10x-21}+1}{\dfrac{x^2+x-6}{10x^2+9x-63}}$.

24. $2a+\dfrac{2}{3a+\dfrac{1}{4a-\dfrac{1}{a}}}$.

26. $\dfrac{4}{1+\dfrac{4}{1+\dfrac{4}{1-x}}}$.

25. $\dfrac{1+\dfrac{4mn}{(m-2n)^2}}{1-\dfrac{4mn}{(m+2n)^2}}\cdot\dfrac{1-\dfrac{2n}{m}}{1+\dfrac{2n}{m}}$.

27. $\dfrac{\dfrac{1}{a-b}-\dfrac{1}{a+b}-\dfrac{1}{b^2-a^2}}{\dfrac{2b+1}{a-b}}$.

28. $\dfrac{\dfrac{m^3-n^3}{m^3+n^3}\cdot\dfrac{(m+n)(m-p)}{m^2-mp-mn+np}}{\left(\dfrac{1}{n^2}+\dfrac{1}{mn}+\dfrac{1}{m^2}\right)\div\left(\dfrac{1}{n^2}-\dfrac{1}{mn}+\dfrac{1}{m^2}\right)}$.

29. $\dfrac{\dfrac{1+3a}{1-3a}-\dfrac{1-3a}{1+3a}}{\dfrac{1-3a}{1+3a}+\dfrac{1+3a}{1-3a}}$.

30. $\dfrac{\dfrac{1}{np}+\dfrac{1}{mp}-\dfrac{1}{mn}}{\dfrac{(m-p)^2-n^2}{\dfrac{1}{np}-\dfrac{1}{mp}-\dfrac{1}{mn}}}$.

31. $\dfrac{\dfrac{x^2+mx+nx+mn}{x^2+nx-px-np}}{\dfrac{x^2+mx-nx-mn}{x^2-nx+px-np}}$.

32. $\dfrac{1+c^3}{1-\dfrac{c}{1+\dfrac{c}{1-c}}}-\dfrac{1-8c^3}{1+\dfrac{2c}{1-\dfrac{2c}{1+2c}}}$.

33. $\dfrac{\left(\dfrac{x+y}{x-y}\right)^2+\left(\dfrac{x-y}{x+y}\right)^2}{\left(\dfrac{x-y}{x+y}\right)^2-\left(\dfrac{x+y}{x-y}\right)^2}$.

34. $\left[\dfrac{\dfrac{m+n}{m-n}+\dfrac{m^2+n^2}{m^2-n^2}}{\dfrac{m-n}{m+n}-\dfrac{m^2+n^2}{m^2-n^2}}\right]^2$.

35. $$\left[\frac{\dfrac{m^2+n^2}{m}-2n}{\dfrac{1}{n^2}-\dfrac{1}{m^2}}\right] \div \frac{m^3-n^3}{m^3+n^3}.$$

36. $$\frac{\dfrac{1}{(m-n)(m+p)}+\dfrac{1}{(m+n)(m-p)}}{\dfrac{1}{(m-n)(m+p)}-\dfrac{1}{(m+n)(m-p)}}.$$

37. $$\frac{\dfrac{3x-1}{3}+\dfrac{x}{4}}{4\frac{2}{3}-\frac{3}{4}(x+\frac{20}{3})+2x}.$$

38. $$\frac{1+\dfrac{c+d}{c-d}}{1-\dfrac{c+d}{c-d}} \div \frac{1+\dfrac{c^2+d^2}{c^2-d^2}}{1-\dfrac{c^2+d^2}{c^2-d^2}}.$$

39. $$\frac{\dfrac{(m+n)^2}{2mn}+1}{\dfrac{4mn}{m^2-4mn+n^2}} \div \left(\frac{m^4+n^4}{mn}-14mn\right).$$

40. $$\left[\frac{\dfrac{1}{a}-\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}} \times \frac{\dfrac{1}{b^2}-\dfrac{1}{ab}+\dfrac{1}{a^2}}{\dfrac{1}{b^2}-\dfrac{2}{ab}+\dfrac{1}{a^2}}\right] \frac{b^2-a^2}{a^4b^4}.$$

41. $$\frac{\left[\dfrac{4b(a+2b)}{a-2b}+a+2b\right]\left[\dfrac{(a^3-8b^3)^2}{a^4-8a^2b^2+16b^4}\right]}{\dfrac{a^2+2ab+4b^2}{a+2b}}.$$

42. $$\frac{\dfrac{(m^3+n^3)(m-n)}{(m^3-n^3)(m+n)}}{\dfrac{m^4-n^4}{(m-n)(m^3+n^3)}} \div \left[\frac{m^2-mn+n^2}{(m-n)^3} \cdot \frac{m^2-2mn+n^2}{m^2+mn+n^2}\right].$$

43. $\left[\dfrac{\left(\dfrac{x+1}{x-1}\right)^2-\left(\dfrac{x+1}{x-1}\right)+1}{\left(\dfrac{x+1}{x-1}\right)^2-\left(\dfrac{x+1}{x-1}\right)-1}-1\right]\left[\dfrac{\left(\dfrac{x+1}{x-1}\right)^2-\left(\dfrac{x+1}{x-1}\right)-1}{4}\right]$.

44. $\left[\dfrac{1-\dfrac{1}{a^2}}{1+\dfrac{1}{a^2}}\right]\div\left[\dfrac{1-\dfrac{1}{a}}{1+\dfrac{1}{a}}\times\dfrac{\left(a-\dfrac{1}{a}\right)\left(1+\dfrac{1}{a}\right)}{a+\dfrac{1}{a}}\right]$.

45. Find the value of $\dfrac{1}{a}-1$ when $a=x+1$.

46. Find the value of $\dfrac{a^2-b^2}{a^2+b^2}$ when $a=x+1$ and $b=x-1$.

47. Find the value of $\dfrac{(m-1)^2-(m^2-1)}{(1-m)^2}$ when $m=a+1$.

48. Find the value of

$$\frac{a+m}{a-m}+\frac{a-m}{a+m}-\frac{3am}{a^2-m^2} \text{ when } m=\frac{3a}{2}.$$

49. When $m=\dfrac{a}{b}$ and $n=\dfrac{b}{a}$ find value of $\dfrac{m^2+mn-2n^2}{m^2-mn-2n^2}$.

50. If $a=\dfrac{x-1}{x+1}$ and $b=\dfrac{x+1}{x-1}$, find value of $\dfrac{a+b}{a-b}+1-\dfrac{a-b}{a+b}$.

51. If $x=a-2$, find value of $\dfrac{x+\dfrac{1}{x+1}-1}{\dfrac{1}{x-1}+1}$.

52. If $a=m$ and $b=n$, show that $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}=\dfrac{4mn}{m^2-n^2}$.

53. If $a=\dfrac{1+c}{1-c}$ and $c=\dfrac{1+m}{1-m}$, find value of a in terms of m and reduce to simplest form.

54. If $x=\dfrac{ab}{a+b}$ and $y=\dfrac{ab}{a-b}$; show that $\dfrac{x^2-y^2}{x^2+y^2}=-\dfrac{2ab}{a^2+b^2}$.

FRACTIONAL EQUATIONS

I. NUMERICAL

Exercise 33

Solve:

1. $\frac{2x}{3}-\frac{x}{5}=\frac{x}{2}-\frac{1}{6}$.

2. $\frac{5x}{3}-\frac{2}{3}=\frac{2x}{5}+\frac{3}{5}$.

3. $\frac{x}{5}+\frac{x}{3}=\frac{7x}{15}-1$.

4. $\frac{2x}{3}-\frac{x}{2}=\frac{x}{4}+\frac{1}{3}$.

5. $\frac{2x}{5}+\frac{x}{8}-\frac{x}{4}-\frac{11}{40}=0$.

6. $\frac{x+1}{2}+\frac{x+3}{4}=2$.

7. $\frac{2x+3}{11}+\frac{x-1}{3}=2$.

8. $\frac{5x-1}{3}+5=\frac{7x+2}{2}$.

9. $\frac{4x+1}{3}+\frac{2x-1}{2}=\frac{3x+3}{2}$.

10. $\frac{5x+1}{4}+\frac{3x-2}{2}=\frac{1+8x}{2}$.

11. $\frac{x+1}{5}-\frac{x-1}{2}=\frac{3-x}{3}$.

12. $\frac{2(x+3)}{5}-\frac{3(x+1)}{2}=0$.

13. $\frac{1}{2}(x+2)=\frac{1}{3}(x-3)$.

14. $\frac{1}{3}(x-1)-\frac{2}{5}(x+1)=0$.

15. $\frac{2}{3}(x+1)-\frac{3}{4}(x+2)=\frac{1}{6}(x+1)$.

16. $(x+\frac{1}{2})(x-\frac{1}{3})=x^2$.

17. $\frac{x}{2}(2x+1)+2=\frac{x}{3}(3x-2)$.

18. $\frac{3x}{5}-\frac{1}{4}(x+3)=\frac{x+5}{2}-3$.

19. $x-\{3x-\frac{1}{2}(x+1)\}=0$.

20. $\frac{1}{2}[x-(2x+\frac{1}{3})]=\frac{1}{3}(x+\frac{1}{4})$.

21. $\frac{x+5}{3}+\frac{1}{2}(2x-1)=\frac{x+\frac{1}{2}}{2}$.

22. $\frac{x+2}{6}-\frac{2}{3}(x+\frac{1}{2})-(x-\frac{1}{3})=0$.

23. $\frac{\frac{1}{2}-x}{3}+12=\frac{\frac{2x}{3}-1}{2}$.

24. $\frac{(x-2)^2}{3}+\frac{(x-3)^2}{2}=\frac{1}{6}(5x^2-1)$.

25. $2\frac{1}{3}-\frac{2}{5}(x^2+3)=\frac{10x}{3}+1-\frac{2x^2}{5}$.

26. $\frac{(x+1)}{3}-\frac{1}{3}\frac{(x+1)}{2}=0$.

27. $\frac{2}{x+1}=\frac{3}{x+2}$.

28. $\frac{3}{x+1}+2=\frac{2x}{x-1}$.

29. $\frac{x+1}{x-1}=\frac{x-1}{x+1}$.

30. $\frac{2x-1}{x+2}-1=\frac{x-3}{x-2}$.

31. $\frac{2x-1}{4x+3}=\frac{3x+1}{6x+1}$.

32. $\frac{3x+2}{4x}+\frac{1}{4}=\frac{x-1}{x+1}$.

33. $\frac{3x-1}{9x+2}=\frac{2x-3}{6x+5}$.

34. $\frac{1}{x+1}+\frac{1}{x-1}=1-\frac{x^2}{x^2-1}$.

35. $\frac{2}{x-2}+\frac{3}{x+2}=\frac{x}{x^2-4}$.

36. $\frac{3}{3x+1}-\frac{2}{1-3x}+1=\frac{9x^2}{9x^2-1}$.

37. $\frac{1}{x^3+1}=\frac{1}{x+1}-\frac{x}{x^2-x+1}$.

38. $\frac{2}{x^2+3x+2}+\frac{1}{(x+2)^2}=0$.

39. $\frac{x^2+x+1}{x+1}=\frac{x^2-x+1}{x-1}+\frac{x}{1-x^2}$.

40. $\frac{x}{3x+6}-\frac{5x^2}{6x^2-24}=-\frac{x}{2x-4}$.

41. $\frac{3}{x^2+3x+2}+\frac{2}{x^2+4x+3}=\frac{1}{x^2+5x+6}$.

42. $\frac{3x+1}{2x^2+x-1}-\frac{x+5}{x^2-x-2}=\frac{x+4}{2x^2-5x+2}$.

43. $\frac{1}{x+2}-\frac{1}{x+3}=\frac{1}{x+4}-\frac{1}{x+5}$.

44. $\frac{1}{x+1}+\frac{2}{x^2-1}=\frac{3}{1-x}$.

45. $\frac{2x-3}{x^2-1}-\frac{x}{1-x}=\frac{x}{x+1}$.

46. $\frac{3}{2x+1}=\frac{4}{4x^2-1}+\frac{2}{1-2x}$.

47. $\frac{x+3}{x-2}-\frac{2}{x^2-4}=\frac{x-1}{2+x}$.

48. $\frac{3}{x-3}-\frac{2}{1-2x}=\frac{5}{2x^2-7x+3}$.

49. $\frac{4x+1}{x+2}=\frac{3}{x^2-4}-\frac{1+4x}{2-x}$.

50. $\frac{x+1}{x+5}-\frac{1-x}{5-x}-\frac{1}{25-x^2}=0$.

51. $\frac{5}{2(2x+5)}-\frac{4}{2x^2+9x+10}-\frac{3}{3x+6}=0$.

52. $\frac{x}{x-3}+\frac{1}{2(x^2-x-6)}=\frac{2}{x+2}+1$.

53. $\frac{x}{6x-6}+\frac{2x}{3-3x}=-\frac{x}{2x+2}$.

54. $\frac{2x-1}{3x+4}-\frac{4x-1}{6x-1}-1=\frac{-18x^2}{21x-4+18x^2}$.

55. $\frac{2}{x+3}-\frac{3}{2(x+3)}+\frac{4}{3(x+3)}=\frac{1}{4}$.

56. $\frac{4x^2-2x+1}{2x-1}+\frac{4x^2+2x+1}{2x+1}=4x$.

57. $\frac{3x-2}{6}+\frac{2x-1}{2x+3}=\frac{x}{2}$.

58. $\frac{3x+5}{18}-\frac{2x-1}{x+5}=\frac{2x-3}{12}$.

59. $\frac{x}{3}-\frac{4x+5}{2x+4}=\frac{2x-1}{6}$.

60. $\frac{2x+13}{8}-\frac{6x-1}{3x-9}=\frac{3x+2}{12}$.

61. $\frac{x+\frac{1}{2}}{6}=\frac{10}{9x-36}+\frac{x-\frac{1}{3}}{6}$.

62. $\frac{x+3\frac{1}{2}}{9}-\frac{x-2}{3x+11}=\frac{x-1}{9}-\frac{1}{18}$.

63. $\frac{x-1}{x-2}-\frac{x-3}{x-4}=\frac{x-4}{x-3}-\frac{x-2}{x-1}$.

64. $\frac{\frac{2x-3}{2}}{5}-\frac{\frac{3x-1}{4}}{3}-\frac{\frac{x+1}{5}}{4}=\frac{1}{3}$.

65. $\frac{2x-3}{\frac{2}{5}}-\frac{3x-1}{\frac{4}{3}}-\frac{x+1}{\frac{5}{4}}=\frac{1}{4}$.

II. LITERAL

Exercise 34

Solve:

1. $3x+5a=x+8a.$
2. $2x+4a=3x+3a.$
3. $3x+2a=4x+a.$
4. $3ax+4a=ax+10a.$
5. $3ax=a(x+a).$
6. $4ax=b(x+a).$
7. $2(x+a)=3(x-a).$
8. $(x+a)^2=(x-a)^2+4a.$
9. $(x-a)^2=(x-b)^2+a^2.$
10. $(a-b)x+(a+b)x=a^2.$
11. $(a+b)x+(a-b)x=a^2+cx.$
12. $(x+a)(x-m)=(x-a)(x+m).$
13. $mx(x+m)+nx(x-m)=(m+n)x^2+m-n.$
14. $ab-(a-2b)x=(2a-b)x-1.$
15. $(x+2a+b)^2-(x+2a)^2=b(8a+5b).$
16. $\frac{x}{a}+\frac{x}{2a}=3.$
17. $\frac{x}{a}-\frac{2x}{3a}=\frac{1}{2}.$
18. $\frac{mx}{n}+\frac{nx}{m}=m^2+n^2.$
19. $\frac{1}{m}+\frac{1}{x}=\frac{1}{n}-\frac{1}{x}.$
20. $cx+a+\frac{x}{c}=\frac{x}{a}.$
21. $\frac{1}{2}\left(\frac{x}{a}+1\right)=\frac{1}{3}\left(\frac{x}{a}-1\right).$
22. $\frac{x+m}{x-n}=\frac{3}{4}.$
23. $\frac{4}{m+x}=\frac{3}{m-x}.$
24. $\frac{x+d^2}{x+c^2}=\frac{c}{d}.$
25. $\frac{m+1}{m-1}=\frac{m+x}{m-x}.$
26. $m-\frac{n(a-x)}{2a-x}-a=0.$
27. $\frac{9}{m+n-x}=\frac{5}{m-n+x}.$

28. $\dfrac{m+n}{x-1}=\dfrac{a-n}{x+1}$.

29. $\dfrac{x}{m}-\dfrac{m}{m-n}=\dfrac{x}{m+n}$.

30. $\dfrac{m+x}{m-x}=\dfrac{x^2}{m^2-x^2}$.

31. $\dfrac{ab+x}{ab-x}=\dfrac{x^2}{x^2-a^2b^2}$.

32. $\dfrac{x+n}{x+2m}=1-\dfrac{3m}{3x+4m+2n}$.

33. $\dfrac{2x-a}{x-a}=\dfrac{x-a}{x+a}+1$.

34. $\dfrac{2x-a}{x+b}-1=\dfrac{x+a}{x-b}$.

35. $\dfrac{\dfrac{m+x}{2}}{\dfrac{m-x}{2}}=\dfrac{1}{m}$.

36. $\dfrac{1-\dfrac{x}{m}}{1+\dfrac{x}{m}}=\dfrac{1+\dfrac{x}{n}}{1-\dfrac{x}{n}}$.

37. $\dfrac{\dfrac{m}{m+1}}{\dfrac{2}{x-1}}=\dfrac{\dfrac{m}{m-1}}{\dfrac{4}{x+1}}$.

38. $\dfrac{3x-a}{x+a}=2-\dfrac{b-x}{b+x}$.

39. $\dfrac{5x-a}{2x-a}=\dfrac{a-10x}{a-4x}$.

40. $\dfrac{x}{m+n}+\dfrac{a}{n-m}=\dfrac{nx}{m^2-n^2}$.

41. $\dfrac{\frac{1}{5}a-\frac{2}{3}x}{\frac{1}{8}a+\frac{3}{2}x}=\dfrac{\frac{1}{2}a+\frac{2}{3}x}{\frac{1}{3}a-\frac{3}{2}x}$.

42. $\dfrac{x+a}{x-a}=\dfrac{x+a+1}{x-a-2}$.

43. $\dfrac{m-2x}{m+x}-\dfrac{m+4x}{m-x}-2=0$.

44. $\dfrac{x+a}{x+c}=\dfrac{x+c}{x+a}$.

45. $\dfrac{x+5}{x-5}-\dfrac{a+b}{a-b}=0$.

46. $\dfrac{c+2x}{c-2x}=-\dfrac{4x^2}{4x^2-c^2}$.

47. $\dfrac{m-\dfrac{x}{m-n}}{m+\dfrac{x}{m-n}}-1=\dfrac{m}{n}$.

48. $\dfrac{m}{x-m}-\dfrac{n}{x-n}=\dfrac{m-n}{x}$.

49. $\dfrac{2x-m}{2x+m}-\dfrac{x+2m}{x-2m}+\dfrac{5m}{x}=0$.

SIMULTANEOUS EQUATIONS

I. NUMERICAL

Exercise 35

Solve:

1. $5x + y = 11$, $3x + 2y = 1$.

2. $x + 2y = 8$, $3x - y = 3$.

3. $2x + 3y = 16$, $3x + 2y = 14$.

4. $x + 2y = 3$, $3x - y = 16$.

5. $x + 3y = 5$, $3x + 4y = 0$.

6. $x - 4y = 7$, $4x - y = 13$.

7. $2x - 13y = 7$, $3x - 21y = 9$.

8. $2x + 3y = 4$, $6x - y = 1$.

9. $5x - y = 16$, $x = y$.

10. $x + 8y = -20$, $3x + 4y = 0$.

11. $x - y = 0$, $4x - 5y + 2 = 0$.

12. $4x - y = 10$, $7y - 2x = 12$.

13. $y - x - 1 = 3$, $x - 5 = -y$.

14. $3x = -2y$, $x = 35 + 11y$.

15. $5x - 3y - 72 = 5y$, $x - 1 = 15y$.

16. $5x + 3y = 102$, $7y + 3x = 104$.

17. $\frac{x}{2} + \frac{y}{3} = 5$, $\frac{x}{6} + \frac{y}{3} = 3$.

18. $\frac{x}{2} + \frac{y}{3} = -2$, $\frac{x}{3} - \frac{y}{2} = -\frac{5}{6}$.

19. $\frac{x}{5} - \frac{y}{4} = 1$, $\frac{x}{2} + y = 9$.

20. $\frac{x}{3} - \frac{y}{4} = \frac{7}{4}$, $\frac{x}{5} + \frac{y}{3} = -\frac{2}{5}$.

21. $\frac{x}{2} = \frac{y}{3}$, $\frac{x}{6} + \frac{y}{3} = 5$.

22. $\frac{1}{x} + \frac{1}{y} = -1$, $\frac{3}{x} - \frac{2}{y} = \frac{9}{2}$.

23. $\frac{x}{2}+\frac{y}{5}=4,$ $\frac{x}{3}-\frac{y}{4}=\frac{3}{4}.$

24. $\frac{x}{3}-\frac{y}{5}=9,$ $\frac{x}{2}+\frac{y}{3}=-13.$

25. $\frac{2x}{3}-\frac{y}{7}=3,$ $\frac{x}{2}-\frac{2y}{3}=-1\frac{2}{3}.$

26. $\frac{3x}{2}+\frac{4y}{3}=-3\frac{5}{12},$ $\frac{2x}{3}-\frac{y}{3}=\frac{1}{3}.$

27. $\frac{5x}{14}-y=\frac{x}{7},$ $\frac{3y}{5}-\frac{x}{8}-\frac{1}{20}=0.$

28. $\frac{2x+3}{7}-\frac{5y-3}{4}=-2,$ $\frac{1}{3}(14y-9)=\frac{16x-23}{5}.$

29. $\frac{x+3}{2}-\frac{y-2}{3}=2,$ $\frac{3x+1}{4}-\frac{y+1}{3}=1.$

30. $\frac{6y-1}{3}+\frac{3x+1}{2}=\frac{5}{3},$ $\frac{5x-1}{4}-\frac{y+3}{7}+\frac{1}{3}=0.$

31. $\frac{x+2}{3}-\frac{y-2}{2}=\frac{3(y-4x)}{4},$ $\frac{x-3}{2}-5=\frac{y+5}{3}-3(y-x).$

32. $x-\frac{2y+1}{5}=2+2(x-y),$ $\frac{x}{3}-\frac{y}{2}=\frac{3(x+y+\frac{5}{2})}{4}.$

33. $\frac{x}{2}-8=\frac{y}{4}+12,$ $\frac{x+y}{5}-35-\frac{2y-x}{4}=-\frac{x}{3}.$

34. $\frac{x+3y-2}{3}=\frac{4y-x+5}{5},$ $\frac{3x-y+7}{10}=\frac{2x+3y+1}{12}.$

35. $\frac{5}{4x+8y+1}=\frac{2}{7x-2y-1},$ $\frac{2x+4y}{3}=\frac{5x+6y}{6}-\frac{4y}{3}.$

36. $\frac{12x+y}{7}=\left(2x-\frac{y}{2}\right),$ $\frac{3x+\frac{4}{5}}{-2y}=\frac{14}{5}.$

37. $\frac{x+y}{x-y}=4,$ $\frac{\frac{y}{4}+\frac{x}{2}}{3}-\frac{\frac{x}{5}+\frac{y}{3}}{5}=\frac{41}{60}.$

38. $\frac{3(y+x)}{4}=\frac{4(y-x)}{\frac{4}{3}}$,

$\frac{4x-3(x-y)}{3}=6\frac{5}{21}-\frac{x-2(x+y)}{7}$.

39. $(x+3)(y-1)=(x-3)(y+1)$,

$xy-\frac{x-y}{2}+\frac{4(x+y)}{3}=(x-3)(y+1)+11\frac{2}{3}$.

40. $\frac{4x-3}{2}+\frac{2x-3}{3x-2y}=\frac{6x-1}{3}$,

$\frac{5y-1}{3}-\frac{x+1}{3x-y}=\frac{15y-10}{9}$.

41. $\frac{2x}{3}-\frac{3y}{5}-\frac{x+2y}{4}=3-\frac{5x-6y}{4}$,

$\frac{x}{2}+y-\frac{3x-y}{5}=-5+\frac{x}{15}$.

42. $\frac{2x-y+3}{3}-\frac{x-2y+3}{4}-4=0$,

$\frac{3x-4y+3}{4}=4-\frac{4x-2y-9}{3}$.

43. $\frac{1}{x}+\frac{2}{y}=1$,

$\frac{4}{x}-\frac{4}{y}=1$.

45. $\frac{5}{x}+\frac{3}{y}=2$,

$\frac{10}{x}-\frac{3}{y}=1$.

47. $\frac{9}{x}-\frac{3}{y}=2$,

$\frac{1}{x}-\frac{1}{y}=0$.

44. $\frac{1}{x}-\frac{3}{y}=\frac{5}{2}$,

$\frac{2}{x}+\frac{1}{y}=-2$.

46. $\frac{3}{x}+\frac{2}{y}=-\frac{31}{40}$,

$\frac{5}{x}-\frac{10}{y}=\frac{11}{8}$.

48. $\frac{1}{2x}+\frac{1}{3y}=-\frac{5}{12}$,

$\frac{1}{3x}-\frac{1}{2y}=\frac{1}{12}$.

49. $\frac{1}{2x}+\frac{5}{3y}=1,$ $\frac{1}{x}+\frac{1}{y}=\frac{5}{6}.$

51. $\frac{5}{2x}+\frac{7}{3y}=-2,$ $\frac{7}{4x}+\frac{5}{6y}=1.$

53. $\frac{10}{3x}-\frac{5}{2y}=-\frac{5}{36},$ $\frac{5}{8x}+\frac{7}{6y}=\frac{19}{24}.$

50. $\frac{2}{3x}+\frac{3}{2y}=-\frac{22}{3},$ $\frac{3}{2x}+\frac{2}{3y}=-\frac{17}{3}.$

52. $\frac{2}{x}-\frac{5}{y}=-4,$ $\frac{3}{x}-\frac{7}{y}=-6\frac{1}{5}.$

54. $\frac{3}{2x}+\frac{5}{4y}=8,$ $\frac{2}{3x}-\frac{1}{2y}=-\frac{2}{3}.$

55. $\frac{3}{4x}+\frac{2}{3y}=-1\frac{3}{4},$ $\frac{2}{3x}-\frac{2}{3y}=2\frac{2}{9}.$

57. $\frac{4}{3x}+\frac{1}{2y}+3=0,$ $-6y+4x=26xy.$

56. $\frac{2x-3}{3}-\frac{4y+3}{2}=x+y,$ $\frac{3x-5}{2}+\frac{y-1}{3}=x-y.$

58. $\frac{1}{x}\left(\frac{x+y}{3}\right)=\frac{5}{6},$ $y\left(\frac{3x-1}{3}\right)=xy-x+5.$

II. LITERAL

Exercise 36

Solve:

1. $x+3y=7a,$ $5x-2y=18a.$

2. $3x+2y=5a,$ $5x+3y=8a.$

3. $ax+by=1,$ $ax-by=3.$

4. $2ax+3by=1,$ $3ax+2by=2.$

5. $x+y=m,$ $2x-3y=n.$

6. $x+y=m+n,$ $3x-2y=m-n.$

7. $x-y=m-n,$ $nx+my=2mn.$

8. $ax+by=m,$ $cx+dy=n$

9. $x + my = -1,$
$y = n(x + 1).$

10. $mx = ny,$
$x + y = a.$

11. $(c + d)x = (c - d)y,$
$x - a = y.$

12. $\frac{1}{2}(x - y) = 2 - \frac{x + y}{3a},$
$x - y = 0.$

13. $(m + n)x + cy = 1,$
$cx + (m + n)y = 1.$

14. $\frac{a}{x} + \frac{b}{y} = c,$
$\frac{a}{x} - \frac{b}{y} = d.$

15. $\frac{x}{c + d} + \frac{y}{c - d} = \frac{3cd - c^2}{c^2 - d^2},$
$x + y = c.$

16. $\frac{a}{bx} + \frac{b}{ay} = a + b,$
$\frac{b}{x} + \frac{a}{y} = a^2 + b^2.$

17. $\frac{x}{m + n} + \frac{y}{m - n} = \frac{2}{m^2 - n^2},$
$\frac{x}{m - n} + \frac{y}{m + n} = \frac{2}{m^2 - n^2}.$

18. $\frac{x + a}{y + a} = \frac{x - 2a}{y - 2a},$
$\frac{x - a}{y - a} = \frac{x + 3a}{y + a}.$

19. $\frac{(m + n)x + (m - n)y}{m^2 + n^2} = 1,$
$mx - ny = m^2 + n^2.$

20. $\frac{a}{x} + \frac{b}{y} = 0,$
$(a + b)x + (a - b)y = a.$

21. $\frac{x + 1}{a} + \frac{y + 1}{c} = 1,$
$\frac{x + 1}{c} + \frac{y + 1}{a} = 1.$

22. $\frac{x}{a + b} + \frac{y}{a - b} = a,$
$\frac{x - y}{ab} = 1.$

23. $\frac{x + y}{m} + \frac{x - y}{n} = 1,$
$\frac{x - y}{m} - \frac{x + y}{n} = 0.$

24. $\frac{x + y + a}{x - y + 1} = 2a,$
$\frac{y + x - a}{y - x - 1} = 3a.$

25. $\frac{(c + d)x - (c - d)y}{4cd} = 1,$
$\frac{(c - d)x}{(c + d)y} = 1.$

26. $\frac{x - m}{p - m} + \frac{y - n}{p - n} = 1,$
$\frac{x + m}{p} + \frac{y - m}{m - n} = \frac{m}{p}.$

III. THREE OR MORE UNKNOWN QUANTITIES

Exercise 37

Solve:

1. $3x + y - 2z = 1,$
   $2x - 3y + z = -1,$
   $4x - 2y + 3z = 14.$

2. $x + 3y + z = 1,$
   $2x + y - 3z = 1,$
   $3x + 2y - 2z = -2.$

3. $2x + 3y - 5z = 0,$
   $3x - 4y - 2z = -3,$
   $2y - 3x + 8z = 7.$

4. $2x + 3y + 4z = 12,$
   $3x - 4y + 5z = 2,$
   $4x + 5y + 6z = 24.$

5. $3x - y + 2z = -11,$
   $3y + 2x - z = -12,$
   $3z + x + 2y = -20.$

6. $2x + y - 10z = 20,$
   $-y - 5z + 3x = 15,$
   $15 + 2x + 3y = z.$

7. $x + y = z + 3,$
   $y = 3x - 8,$
   $z - x = 4.$

8. $x + 2y = 25,$
   $y - 2z = 0,$
   $x + 3z = 20.$

9. $x + y = 2a,$
   $x + z = 3a,$
   $y + z = 2a.$

10. $x + 2y + z = a,$
    $x + y + 2z = b,$
    $y + z + 2x = c.$

11. $x + 20 = 2(y - z),$
    $y + 20 = 3(z - x),$
    $z + 20 = 2(x - y).$

12. $x + y + z = a + b,$
    $x + y - z = a - b,$
    $y + z - x = c - a.$

13. $\frac{x}{3} + \frac{y}{2} + \frac{z}{4} = \frac{53}{12},$
    $\frac{x}{4} + \frac{y}{3} - \frac{z}{2} = \frac{13}{12},$
    $\frac{x}{2} - \frac{y}{4} + \frac{z}{3} = \frac{5}{2}.$

14. $\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z = 23,$
    $\frac{1}{3}x + \frac{1}{4}y + \frac{1}{2}z = 28,$
    $\frac{1}{4}x + \frac{1}{2}y + \frac{1}{3}z = 27.$

15. $\frac{x}{2+y}=\frac{2}{5}$,

$\frac{y}{3+z}=\frac{3}{7}$,

$\frac{z}{4+x}=\frac{2}{3}$.

16. $ay+bx=1$,

$cx+az=1$,

$bz+cy=1$.

17. $\frac{1}{3x}+\frac{1}{2y}+\frac{1}{z}=2\frac{5}{6}$,

$\frac{1}{2x}+\frac{1}{3y}-\frac{1}{z}=-2$,

$\frac{1}{x}+\frac{1}{y}+\frac{1}{2z}=7$.

18. $x+y+z=33$,

$x+z+u=35$,

$x+y+u=34$,

$y+z+u=36$.

19. $\frac{1}{x}+\frac{1}{y}=\frac{5}{6}$,

$\frac{1}{y}+\frac{1}{z}=\frac{7}{12}$,

$\frac{1}{x}+\frac{1}{z}=\frac{3}{4}$.

20. $\frac{1}{x}+\frac{2}{y}-\frac{1}{z}=5$,

$\frac{2}{x}-\frac{4}{y}+\frac{3}{z}=-3$,

$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=\frac{1}{2}$.

21. $\frac{1}{x}+\frac{1}{y}=m$,

$\frac{1}{x}+\frac{1}{z}=n$,

$\frac{1}{y}+\frac{1}{z}=p$.

22. $\frac{15}{x}+\frac{8}{y}-\frac{6}{z}=4$,

$\frac{9}{x}-\frac{4}{y}+\frac{4}{z}=4$,

$\frac{3}{x}+\frac{6}{y}-\frac{1}{z}=2$.

23. $\frac{xy}{x-y}=7\frac{1}{2}$,

$\frac{xz}{x-z}=1\frac{1}{4}$,

$\frac{yz}{y-z}=1\frac{1}{2}$.

24. $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{3}{2},$

$\frac{a}{x}+\frac{b}{y}-\frac{c}{z}=\frac{1}{2},$

$\frac{a}{x}-\frac{b}{y}-\frac{c}{z}=-\frac{1}{2}.$

25. $mx+ny=a(m+n),$
$mx+az=n(a+m),$
$ny+az=m(a+n).$

26. $x+y=22,$
$y+z=18,$
$z+w=14,$
$w+u=10,$
$u+x=16.$

IV. PROBLEMS IN SIMULTANEOUS EQUATIONS

Exercise 38

1. A man purchased 20 acres of land for $1640. Part of it was bought for $90 an acre and the remainder for $50 an acre. How many acres were there in each portion?

2. A man and a boy together weigh 230 pounds, and twice the man's weight is 60 pounds more than 3 times the boy's weight. Find the weight of each.

3. Three horses and 4 cows can be bought for $610, but at the same rates it takes $720 to purchase 4 horses and 3 cows. Find the price of each per head.

4. If half of A's money is added to B's money, the sum is $170; but if half of B's is added to A's money, the sum is $160. How much money has each?

5. In 10 hours A walks 1 mile more than B walks in 8 hours. In 5 hours B walks $5\frac{1}{2}$ miles less than A walks in 7 hours. How many miles does each walk per hour?

6. If the numerator of a certain fraction is subtracted from the denominator, the remainder is 21; but if the denominator is subtracted from 8 times the numerator, the remainder is -7. Find the fraction.

7. In a certain town meeting 312 voters were present, and a motion was carried by a majority of 8 votes. How many voted for and against the motion?

8. Two men had together \$100, and if the first had given \$10 to the second, each would then have had the same amount. How much had each originally?

9. If 2 is added to both numerator and denominator of a certain fraction, the resulting fraction is $\frac{4}{5}$. If 1 is subtracted from both numerator and denominator, the new fraction is $\frac{1}{2}$. Find the original fraction.

10. A boatman can row 20 miles down a stream and back in 10 hours, the current being uniform. He can row 2 miles upstream in the same time that he can row 3 miles downstream. Find his rate per hour both down and upstream.

11. If the width of a field were increased 1 rod and the length 2 rods, the area would be 34 square rods greater; but if the width were decreased 2 rods and the length increased 3 rods, the area would be unchanged. Find the length and width of the field.

12. Find a fraction such that if you double the numerator and add 3 to the denominator the result is $\frac{2}{3}$; but if you add 3 to the numerator and double the denominator the result is $\frac{5}{6}$.

13. Two sheep-owners met. A said, "Sell me 4 of your sheep, and I shall have twice as many as you have." B said, "No; sell me 1 of yours, and we shall each have the same number." How many had each?

14. When a certain number of two digits is doubled and increased by 4, the result is the same as if the digits had been reversed and this number decreased by 22. The number is 2 less than 3 times the sum of its digits. Find the number.

15. If I divide a certain number by 3 more than the sum of its two digits, I get a quotient of 3 and a remainder of 8. But if I reverse the order of the digits and divide by twice the sum of the digits in the same reversed order, my quotient is 3 and remainder 11. Find the number.

16. A boy bought 5 apples and 3 oranges for 25 cents, 4 oranges and 5 pears for 35 cents, 2 pears and 7 apples for 20 cents. Find the prices paid for each apple, orange, and pear.

17. Find 3 numbers such that if each be added to $\frac{1}{3}$ the sum of the others, the results will be 32, 28, and 30 respectively.

18. The sum of the three digits of a number is 12. The hundreds' digit is one half the sum of the other two, and the units' digit is $\frac{1}{7}$ the number composed of the other two in the original order. Find the number.

19. Three boys together weigh 300 pounds. Half the sum of the weights of the first and the third equals the weight of the second. The sum of the weights of the second and the third divided by the difference between the weights of the third and the first gives a quotient of 5 and a remainder of 20. Find the weight of each.

20. A and B together can do a certain piece of work in 3 days, A and C the same work in 4 days, B and C the same work in 6 days. How long will it take each alone to do the work? How long will it take all working together?

21. Some books were divided among 3 boys, so that the first had 12 less than half of all, the second 1 less than half the remainder, and the third 17. Find the number each received.

22. A boy has 100 pieces of silver. The value of the quarters is 3 times the value of the dimes, and the sum of the values of the half dollars and dimes divided by the difference of the values of the quarters and half dollars is $\frac{5}{8}$. Find the number of dimes, quarters, and half dollars.

INVOLUTION AND EVOLUTION

I. MONOMIALS

Exercise 39

Write the value of:

1. $(2a)^2$.
2. $(3a^2)^2$.
3. $(-2a^2)^2$.
4. $(3a)^4$.
5. $(-3a)^4$.
6. $-(3a)^4$.
7. $(-3a)^3$.
8. $(-2a^2)^5$.
9. $(-2mn)^3$.
10. $(-2a^2)^6$.
11. $(3ab^2c)^3$.
12. $(4m^2ny)^4$.
13. $(\frac{1}{2}a^2)^2$.
14. $\left(\frac{2a}{3}\right)^2$.
15. $(-\frac{2}{3}ab)^3$.
16. $\left(\frac{3a^2}{2}\right)^3$.
17. $\left(\frac{2a}{-3bc}\right)^4$.
18. $\left(-\frac{2am}{3cd}\right)^5$.
19. $-\left(\frac{2a}{3c^2}\right)^3$.
20. $-\left(-\frac{2a}{3c^2}\right)^3$.
21. $\left(\frac{2ab}{3c^2d}\right)^4$.
22. $\left(\frac{3ab^2}{-2m}\right)^5$.
23. $-\left(\frac{2a^3d^4}{3m^2n}\right)^4$.
24. $\left(\frac{3a^2c}{4xy^4}\right)^3$.
25. $\left(\frac{2m^2n^3}{3x^4y^2z^5}\right)^3$.
26. $\sqrt{4m^2}$.
27. $\sqrt{16m^2n^4}$.
28. $\sqrt[3]{8c^3d^3}$.
29. $\sqrt[3]{27c^6}$.
30. $\sqrt[3]{64m^6n^3}$.
31. $\sqrt[3]{-27m^6}$.
32. $\sqrt{100x^{10}y^4}$.
33. $\sqrt[3]{-64a^{12}c^9}$.
34. $\sqrt[4]{16m^4n^4}$.
35. $\sqrt[4]{16m^8n^{12}}$.
36. $\sqrt{\frac{4a^2c^2}{9m^2}}$.
37. $\sqrt{\frac{9a^4c^6}{16m^4}}$.
38. $\sqrt{\frac{25x^{10}}{36y^{14}}}$.
39. $\sqrt[3]{\frac{64a^3}{343c^{12}}}$.
40. $\sqrt[5]{\frac{-a^{10}}{32a^5}}$.
41. $\sqrt[4]{16\left(\frac{a^8c^{12}}{m^4n^8}\right)}$.
42. $\sqrt[5]{-32\left(\frac{a^{10}b^{15}}{c^{20}}\right)}$.
43. $\sqrt[3]{-27(a+b)^3}$.
44. $\sqrt[4]{81(m-n)^8}$.
45. $\sqrt[5]{-32(a+1)^5}$.

II. INVOLUTION — BINOMIALS

Exercise 40

Expand:

1. $(a+b)^3$.
2. $(a+b)^4$.
3. $(a+m)^5$.
4. $(a+2)^4$.
5. $(a-3)^5$.
6. $(2a+3)^4$.
7. $(3a-2)^4$.
8. $(2a^2+5)^3$.
9. $(ab-1)^4$.
10. $(a-2)^7$.
11. $(a^2b^2+2c)^3$.
12. $(d^2-4c)^4$.
13. $(d^2-3c)^5$.
14. $(c^3+2c^2)^5$.
15. $\left(a+\frac{2}{c}\right)^3$.
16. $(2a-\frac{2}{3})^4$.
17. $\left(a^2+\frac{2c}{3}\right)^4$.
18. $\left(\frac{ab}{2}-1\right)^5$.
19. $\left(\frac{c}{3}-\frac{1}{2}\right)^5$.
20. $\left(c^2-\frac{c}{2}\right)$.
21. $(a^2+a+1)^2$.
22. $(x^2-3x+2)^2$.
23. $(m^4-n^2-3)^2$.
24. $(a^2+a+1)^3$.
25. $(a^2-a+1)^3$.
26. $(a+c-\frac{1}{2})^3$.
27. $(2a-a^2+1)^3$.

III. EVOLUTION — SQUARE ROOT

Exercise 41

Extract the square root of:

1. $x^4+2x^3+5x^2+4x+4$.
2. $x^4-6x^3+17x^2-24x+16$.
3. $x^6+2x^5+x^4-2x^3-2x^2+1$.
4. $x^4-2x^3+11x^2-10x+25$.
5. $x^6+4x^4-8x^3+4x^2-16x+16$.
6. $4x^4-20x^3+37x^2-30x+9$.

7. $9 - 12x - 2x^2 + 4x^3 + x^4$.

8. $a^6 - 2a^5x + 5a^4x^2 - 6a^3x^3 + 6a^2x^4 - 4ax^5 + x^6$.

9. $12x^3 - 30x + 4x^4 + 25 - 11x^2$.

10. $25x^2y^4 + 2x^4y^2 + x^6 - 8x^3y^3 - 12xy^5 + 36y^6 - 4x^5y$.

11. $x^2 - 2x^5 - 14x + 49 + 14x^4 + x^8$.

12. $\frac{x^4}{4} + x^3 + \frac{4x^2}{3} + \frac{2x}{3} + \frac{1}{9}$.

13. $\frac{x^4}{9} - \frac{4x^3}{3} + \frac{23x^2}{6} + x + \frac{1}{16}$.

14. $\frac{a^2}{c^2} + \frac{4a}{c} + 2 - \frac{4c}{a} + \frac{c^2}{a^2}$.

15. $4 + 4a - a^2 + \frac{a^3}{3} + \frac{11a^4}{12} - \frac{a^5}{3} + \frac{a^6}{9}$.

16. $\frac{a^2}{9} + \frac{9}{a^2} + \frac{6}{a} - 1 - \frac{2a}{3}$.

17. $c^6 - \frac{c^5}{2} + \frac{33c^4}{16} - \frac{c^3}{2} - \frac{c^3}{a} + c^2 + \frac{c^2}{4a} - \frac{c}{a} + \frac{1}{4a^2}$.

Extract, to three terms, the square root of:

18. $1 + 3a$.

19. $1 - 5a$.

20. $a^2 + 9b$.

21. $x^2 + 4x$.

Extract the fourth root of:

22. $81x^4 - 216x^3 + 216x^2 - 96x + 16$.

23. $a^4 - 12a^3b^2 + 54a^2b^4 - 108ab^6 + 81b^8$.

24. $16a^4 + 16a^3x + 6a^2x^2 + ax^3 + \frac{x^4}{16}$.

IV. EVOLUTION—CUBE ROOT

Exercise 42

Extract the cube root of:

1. $a^3+9\,a^2+27\,a+27$.
2. $a^6-15\,a^5+75\,a^4-125\,a^3$.
3. $a^6+6\,a^5+15\,a^4+20\,a^3+15\,a^2+6\,a+1$.
4. $1-9\,a+33\,a^2-63\,a^3+66\,a^4-36\,a^5+8\,a^6$.
5. $60\,x^2+1+240\,x^4+64\,x^6-192\,x^5-160\,x^3-12\,x$.
6. $m^6-3\,m^5+5\,m^3-3\,m-1$.
7. $18\,a^4+90\,a^2+125-3\,a^5-31\,a^3-75\,a+a^6$.
8. $m^3+m^2+\frac{m}{3}+\frac{1}{27}$.
9. $a^6+\frac{3\,a^5}{x^2}+\frac{3\,a^4}{x^4}+\frac{a^3}{x^6}$.
10. $a^3-\frac{3\,a^2b}{2\,c}+\frac{3\,ab^2}{4\,c^2}-\frac{b^3}{8\,c^3}$.
11. $x^6-3\,x^5+2\,x^4-x^3+\frac{2\,x^2}{3}-\frac{x}{3}-\frac{1}{27}$.

Extract the sixth root of:

12. $1-6\,m+15\,m^2-20\,m^3+15\,m^4-6\,m^5+m^6$.
13. $a^6+60\,a^4b^2+240\,a^2b^4+64\,b^6-12\,a^5b-160\,a^3b^3-192\,ab^5$.

V. EVOLUTION—NUMERICAL

Exercise 43

Find the square root of:

1. 3969.
2. 6561.
3. 8464.
4. 10404.
5. 15129.
6. 93636.
7. 1772.41.
8. 2672.89.
9. .986049.
10. .01449616.
11. .01018081.
12. .000104101209.

Find, to four decimal places, the square root of:

13. 8.
14. 14.
15. 175.
16. 2.5.
17. 37.561.
18. .375.
19. $\frac{5}{8}$.
20. 1.0405.
21. .0035.

Find the cube root of:

22. 42875.
23. 250047.
24. 1860867.
25. 12977875.
26. 28652616.
27. 74.618461.
28. .001481544.
29. 34328.125.
30. 20.570824.

Find, to two decimal places, the cube root of:

31. 9.
32. 67.
33. 7.3.
34. $2\frac{1}{4}$.
35. $\frac{3}{8}$.
36. .0042.

Find, to two decimal places, the value of:

37. $\sqrt{5}$.
38. $\sqrt{.5}$.
39. $\sqrt{40}$.
40. $\sqrt{.9}$.
41. $\sqrt{.07}$.
42. $\sqrt{.007}$.
43. $\sqrt{1.12}$.
44. $\sqrt[3]{11.2}$.
45. $\sqrt{.0017}$.
46. $\sqrt[3]{2\sqrt{2}}$.
47. $\sqrt{5}+\sqrt{3}$.
48. $\sqrt{5+\sqrt{3}}$.
49. $\sqrt[3]{10}+\sqrt{7}$.
50. $\sqrt{15}-3\sqrt[3]{18}$.
51. $\sqrt{\dfrac{5+\sqrt{5}}{3}}$.
52. $\sqrt{\dfrac{10+4\sqrt{3}}{\sqrt[3]{2}}}$.
53. $\sqrt{.038+\sqrt{.009}}$.
54. $\sqrt{\sqrt{.5}+\sqrt[3]{.6}}$.
55. $\sqrt{10+\sqrt{10}+\sqrt[3]{10}}$.

REVIEW

Exercise 44

1. Find the H. C. F. of $5\,a^4 - 4\,a^2 - 64$ and $a^4 + a^2 - 20$.

2. Take $x^3 + 3$ from $x^3 - 2\,x^2 + x$, and multiply the remainder by $x\,(x + 3)$.

3. Find four terms of $a^5 \div (a - 2)$.

4. Prove that $\dfrac{(a+b)^2 - (a-b)^2}{4} = ab$.

5. Find the value of $\sqrt{x^3 + y^3 + z^3} - (x - y - z)^3$, when $x = 3$, $y = -3$, and $z = 0$.

6. What is the remainder if $(a - 2)(a - 3)(a^2 - a + 5)$ is divided by $(a - 1)(a + 2)$?

7. A certain divisor is $x^2 + x - 2$ and the corresponding quotient is $x^2 - x - 1$. Find the corresponding dividend.

8. What quotient will result if the sum of $x^3 - 5\,x^2 + 8\,x$ and $2\,x^2 - 5\,x - 1$ is divided by the sum of $3\,x^2 - 2\,x - 3$ and $-2\,x^2 + 4$?

9. Solve $\dfrac{3\,x - 4}{x + 2} + \dfrac{3 - x}{x - 5} = 2$.

10. Prove that

$$\left(a - \frac{c^2 - ab}{a - b}\right)\left(b + \frac{c^2 - ab}{a - b}\right) + \left(\frac{c^2 - ab}{a - b}\right)^2 = c^2.$$

11. If $m=\dfrac{1}{a+1},\ n=\dfrac{2}{a+2},\ p=\dfrac{3}{a+3}$,

find the value of $\dfrac{m}{1-m}+\dfrac{n}{1-n}+\dfrac{p}{1-p}$.

12. What value of x will make the expression

$$3(x+2)-4(x-3)$$

equal to twice the value of x?

13. If $\dfrac{3x+2a}{3a-x}=9$, find the value of $\dfrac{x+2a}{x-a}$.

14. Show that

$$-2x-[4-\tfrac{1}{2}(x+4)-2\{x-3-(x+2)\}-4]-\frac{5x}{2}=0.$$

15. Show that

$$\frac{a(b+c-a)}{(a-b)(c-a)}+\frac{b(c+a-b)}{(b-c)(a-b)}+\frac{c(a+b-c)}{(c-a)(b-c)}=2.$$

16. Simplify $\dfrac{\dfrac{a+4b}{a-4b}+2-\dfrac{4b-a}{4b+a}}{\dfrac{2a}{(a+4b)^2}}\times\left(\dfrac{2}{a}-\dfrac{1}{2b}\right)$.

17. If $a=-2$, find value of $\dfrac{(1-a)(1-2a)}{(1+2a)}-\dfrac{(1+a)(1+2a)}{(1-2a)}$.

18. Solve $\dfrac{a-2x}{2a-x}-1-\dfrac{a+x+1}{x+2a}=0$.

19. Simplify $\dfrac{a-4+\dfrac{6}{a-1}}{a-\dfrac{6}{a-1}}\cdot\dfrac{1-\dfrac{a+5}{a^2-1}}{\dfrac{(a-1)(a-2)}{a+1}}$.

20. Add $4\left[\dfrac{2a}{3}-\dfrac{1}{2}\left(3c-\dfrac{1}{2}\right)\right]$ and $3\left[\dfrac{4a}{5}-\dfrac{1}{3}\left(c+\dfrac{1}{5}\right)-2\right]$.

21. Solve $\frac{mx}{x+n}+\frac{nx}{x+m}=m+n$.

22. Show that $(a+b)(a+b-1)=a(a-1)+2ab+b(b-1)$.

23. Find the value of $\frac{ax+1}{x}-\left[a(x+1)-\frac{a(x^2-1)-x}{x}\right]$ when $x=\frac{a-1}{a}$.

24. If $a=\frac{2b-3}{b^2}$ and $c=\frac{a+1}{a+3}$, find c when $b=-1$.

25. Simplify $\frac{a-[b-c-\{2a-2b-\frac{1}{2}(3c-b)\}]}{a-\frac{b}{c+\frac{1}{a}}}$, when $a=1$, $b=2$, $c=3$.

26. Solve $x-\frac{3x+1}{5}-\frac{\frac{4x}{5}-1}{2}=\frac{3}{10}-\frac{5x+2}{11x+6}$.

27. What must be the value of m in order that

$$6a^4-a^3-11a^2-10a-m$$

may be exactly divisible by $2a^2-3a-1$?

28. Show that $\frac{x(x-a)}{c}-\frac{c(c+a)}{x}=a$ when $a=x-c$.

29. Given that $m=\frac{1-a^2}{1+a^2}$ and $n=\frac{2a}{1+a^2}$, prove that $m^2+n^2=1$.

30. Prove that $\frac{m}{3}$ satisfies the equation

$$\frac{4x-m}{2x-m}-\frac{x+m}{x-m}=1.$$

31. Solve $(x+3)(y-2)=(x-5)(y+4)+16$,

$$(x-y)(x+1)=1-x(y-x).$$

32. If $x=\frac{a+b}{2}$ and $a=\frac{m-2b}{2}$, find x in terms of m.

33. Solve $a(a-x)=b(x+y-a)$,

$$a(y-b-x)=b(y-b).$$

34. When $x=2$, find the value of

$$3x-\left[2-\frac{1}{2}\left(3x+\frac{4}{3}\right)-\left\{x\left(2-\frac{x}{2}\right)-2\left(x-\frac{1}{3}\right)\right\}-\frac{x^2}{2}\right].$$

35. Solve $(a-2b)x+a^2+b^2=(2a-b)x-2ab$.

36. What must be added to $2[1-3x\{1-2x(1-5x)\}]$ to produce $-5-3x(1-2x)^2$?

37. If $x=\frac{m-1}{n}$ and $m=\frac{n+1}{2}$, find value of x in terms of n.

38. What must be the value of m in order that $x=2$ may be the solution of $m(x-m)=3(x-3)$?

39. Arrange $(m-2)^4-(m-1)^3+3(2-m)^2$ in ascending powers of m.

40. Solve $\frac{x-m}{x+2n}=\frac{x+m}{x-2m}$.

41. Expand $[m-3n-\{3(m-n)-n\}]^4$.

42. Find the square root of

$$(m-n)^2[(m-n)^2-2(m^2+n^2)]+2(m^4+n^4).$$

43. Solve $\frac{x-m}{cn}+\frac{x-n}{cm}+\frac{x-c}{mn}=\frac{2}{m}+\frac{2}{n}+\frac{2}{c}$.

44. What expression multiplied by itself will give the expression $4x^6 - 12x^5 + 5x^4 + 14x^3 - 11x^2 - 4x + 4$?

45. Expand $(a^2 - 3c)^4$ and extract square root of the result.

46. Divide $1 - \frac{3ab}{(a+b)^2}$ by $1 - \frac{ab}{a^2 - ab + b^2}$ and extract square root of the quotient.

47. Simplify $\sqrt[3]{(a-2c)^6}$.

48. Show that the difference between the squares of any two consecutive numbers is 1 more than double the smaller number.

49. Prove that $\dfrac{a + \dfrac{1}{a-1}}{\dfrac{1}{a} - \dfrac{1}{a^3 - a^2 + a}}$ is a perfect square.

50. Compute $\sqrt{0.4 + \sqrt{.004}}$ to three decimal places.

51. Find the cube root of $[2a - (a + \sqrt{1 - (a-2)})]^6$.

52. If $x = \dfrac{a+1}{a-1}$, find the value of

$$\sqrt{(x+4)(x-2) - 2(x-4)}.$$

53. Simplify $\sqrt[3]{\sqrt{10} - \sqrt{5}}$ to two decimal places.

54. Solve $\dfrac{a - 3x}{b + x} = \dfrac{b - 2x}{a + x} - 1$.

55. Find the square root of

$$\frac{(x^2 - 4x + 3)(x^2 - 9)(x^2 + 2x - 3)}{x^2 + 6x + 9}.$$

56. Find, to two decimal places: $\sqrt{\dfrac{2 + \sqrt{17}}{\sqrt{150}}}$.

57. Solve $15y - 14x = -4xy$; $30y - 18x = 17xy$.

EXPONENTS

I. TRANSFORMATIONS

Exercise 45

Express with fractional exponents:

1. $\sqrt{a}$.
2. $\sqrt[3]{a}$.
3. $\sqrt{a^3}$.
4. $2\sqrt{a^3}$.
5. $5\sqrt[3]{a^2}$.
6. $\sqrt{a^6}$.
7. $\sqrt[3]{a^5}$.
8. $\sqrt{a^n}$.
9. $a\sqrt[n]{c^m}$.
10. $\sqrt[n]{a^{2n}}$.
11. $3\sqrt[n]{a^{mn}}$.
12. $\sqrt{4\,a^3}$.
13. $\sqrt[3]{27\,x^5}$.
14. $\sqrt[3]{mn^2}$.
15. $\sqrt[6]{ab^2c^3d^4}$.
16. $\sqrt[5]{32x^5y^{10}z^{15}}$.

Express with radical signs:

17. $a^{\frac{1}{2}}$.
18. $a^{\frac{1}{3}}$.
19. $a^{\frac{1}{4}}$.
20. $a^{\frac{2}{3}}$.
21. $5\,a^{\frac{1}{2}}$.
22. $ab^{\frac{1}{2}}$.
23. $abc^{\frac{1}{2}}$.
24. $3\,ab^{\frac{1}{2}}c^{\frac{1}{2}}$.
25. $a^{\frac{1}{2}}b^{\frac{2}{3}}$.
26. $a^{\frac{2}{3}}b^{\frac{1}{4}}$.
27. $a^{\frac{2}{3}}b^{\frac{2}{3}}$.
28. $(ab)^{\frac{2}{3}}$.
29. $mn^{\frac{1}{a}}$.
30. $3\,c^m d^{\frac{1}{m}}$.
31. $7\,cd^{\frac{2}{3}}$.
32. $2\,c^{\frac{2}{3}}d^{\frac{1}{3}}$.
33. $a^{\frac{2}{3}}b^{\frac{1}{2}}c^{\frac{3}{2}}$.
34. $2\,a^{\frac{1}{2}}b^{\frac{3}{2}}c^{\frac{5}{2}}$.
35. $4\,c^{\frac{3}{4}}d^{\frac{n}{m}}$.
36. $5\,a^{\frac{m}{n}}b^{\frac{n}{m}}$.

In the following, transfer to denominators all factors having negative exponents:

37. ab^{-1}.
38. $ab^{-1}c^{-2}$.
39. $2\,abc^{-2}$.
40. ab^2c^{-2}.
41. $7\,a^{-1}$.
42. $3\,a^{-1}b^{-1}$.
43. $a^{-1}bc^{-1}$.
44. $a^{-1}b^{-1}c^{-1}$.
45. $2^{-1}a^{-1}$.
46. $3\,a^2b^{-1}$.
47. $4\,a^{-\frac{1}{2}}x$.
48. $3^{-1}m$.
49. $9^{-\frac{1}{2}}cd$.
50. $-2^{-1}a^{-1}bc^{-1}$.
51. $-\frac{2}{3}\,a^{-1}bc$.

Write the following without denominators:

52. $\frac{a^2}{b}$.

53. $\frac{ab}{c}$.

54. $\frac{abc}{mn}$.

55. $\frac{2a}{cd}$.

56. $\frac{3a^2}{c^{-1}}$.

57. $\frac{2a}{b^{-1}c^{-2}}$.

58. $\frac{2mn}{ac^{-1}}$.

59. $\frac{4a^{-2}}{b^{-1}cd^{-1}}$.

60. $\frac{4c}{a^{-1}x^{-\frac{3}{2}}}$.

61. $\frac{3a^{-1}}{x^{-\frac{m}{n}}c^{-2}}$.

Express the following with positive exponents:

62. $3a^{-1}$.

63. $2am^{-2}$.

64. $a^2b^{-1}c$.

65. $mn^{-\frac{1}{2}}$.

66. $2a^{-1}b^2c^{-1}$.

67. $5m^{-1}b^{-3}c$.

68. $ab^{-2}xy^{-3}$.

69. $mn^{-4}y^{-1}z$.

70. $a^{-1}b^{-4}c^{-1}$.

71. $\frac{3}{a^{-1}b}$.

72. $\frac{4a^{-1}}{cd^{-1}}$.

73. $\frac{2ab^{-1}}{3m^{-1}n}$.

74. $\frac{1}{xy^{-3}}$.

75. $\frac{2^{-1}x^2}{a^{-1}}$.

76. $\frac{4\sqrt{x^{-1}}}{a^{-1}}$.

77. $\frac{3^{-2}x^{-1}}{a^{-1}b}$.

78. $\frac{2\sqrt{a^{-2}c^{-1}}}{3mn^{-2}}$.

79. $\frac{4a^{-1}cd^{-3}}{\sqrt{x^{-1}}\sqrt[3]{m^2}}$.

80. $\frac{2^{-1}ab^{-1}c}{3\sqrt{x^{-3}}}$.

Find the numerical value of the following:

81. $4^{\frac{1}{2}}$.

82. $9^{\frac{1}{2}}$.

83. $4^{\frac{3}{2}}$.

84. $9^{\frac{3}{2}}$.

85. $16^{\frac{1}{4}}$.

86. $27^{\frac{1}{3}}$.

87. $27^{\frac{2}{3}}$.

88. $81^{\frac{3}{4}}$.

89. $125^{\frac{1}{3}}$.

90. $(-27)^{\frac{1}{3}}$.

91. $(-64)^{\frac{1}{3}}$.

92. $(-125)^{\frac{2}{3}}$.

93. $4^{-\frac{1}{2}}$.

94. $9^{-\frac{3}{2}}$.

95. $16^{-\frac{5}{4}}$.

96. $(-27)^{-\frac{1}{3}}$.

97. $36^{-\frac{3}{2}}$.

98. $(-32)^{-\frac{3}{5}}$.

99. $\sqrt[3]{-27}$.

100. $\sqrt[3]{(-27)^2}$.

101. $(\sqrt[3]{-27})^2$.

102. $\sqrt[3]{8^{-2}}$.

103. $(\sqrt[4]{16})^3$.

104. $(\sqrt[5]{-32})^6$.

105. $2^{-2} \cdot 3^{-1}$.

106. $3^{-2} \cdot 2^{-2}$.

107. $9^{-\frac{1}{2}} \cdot 27^{\frac{1}{3}}$.

108. $16^{\frac{1}{4}} \cdot 8^{-\frac{2}{3}}$.

109. $81^{-\frac{3}{4}} \cdot \frac{1}{9^{-1}}$.

110. $2^{-3} \div 2^{-1}$.

111. $16^{-\frac{1}{4}} \cdot 8^{\frac{1}{3}}$.

112. $8^{\frac{4}{3}} \cdot \frac{1}{32^{\frac{1}{5}}}$.

113. $9^{-\frac{3}{2}} \cdot 81^{\frac{5}{4}}$.

114. $2^{-2} \cdot 3^2 \cdot 4 \cdot 6^{-1}$.

115. $(2\frac{1}{4})^{\frac{1}{2}}$.

116. $(1\frac{9}{16})^{-\frac{3}{2}}$.

117. $-(-2\frac{1}{27})^{-\frac{2}{3}}$.

118. $(\frac{3}{2}\frac{2}{4}\frac{2}{3})^{-\frac{2}{5}}$.

119. $(-\frac{6}{2}\frac{4}{7})^{-\frac{1}{3}}$.

120. $(\frac{1}{4})^{-\frac{3}{2}} \div (\frac{1}{8})^{\frac{2}{3}}$.

121. $3^{-2} \cdot \frac{5}{6^{-1}}$.

122. $\frac{4^{-\frac{3}{2}} \cdot 9^{\frac{3}{2}}}{27^{\frac{2}{3}}}$.

123. $\frac{16^{-\frac{5}{4}} \cdot 27^{-\frac{4}{3}}}{9^{-\frac{1}{2}} \cdot 64^{-\frac{4}{3}}}$.

124. $(4^{-3} \cdot 3^{-4} \cdot 2^3) \div (16^{-\frac{3}{4}} \cdot 27^{-\frac{2}{3}} \cdot 81^{-\frac{1}{2}})$.

Perform the indicated operations in the following:

125. $a^3 \cdot a^{-1}$.

126. $a^2 \cdot a^{-1}$.

127. $a \cdot a^{-3}$.

128. $a^2 \cdot a^{-2}$.

129. $a^{-4} \cdot a^{-1}$.

130. $a^{-3} \cdot a^0$.

131. $a^{\frac{1}{2}} \cdot a^{\frac{1}{2}}$.

132. $a \cdot a^{\frac{1}{2}}$.

133. $a^{-1} \cdot a^{-\frac{1}{3}}$.

134. $a^{\frac{1}{2}} \cdot a^{-\frac{1}{4}}$.

135. $m^{-2} \cdot m^{\frac{2}{3}}$.

136. $m^{-\frac{2}{3}} \cdot m^{\frac{2}{3}}$.

137. $a^{-4} \div a^2$.

138. $a^{-3} \div a^{-2}$.

139. $a^{-1} \div a^{\frac{2}{3}}$.

140. $a^2x \cdot ax^{-2}$.

141. $am^2n^{-1} \cdot a^2m^{-1}n^2$.

142. $2\,ab^2 \cdot 3\,a^{-1}b^{-1}$.

143. $a^0 + b^0$.

144. $3\,a^0 - (3\,a)^0$.

145. $(a+b)^0$.

146. $(-2)^{-3} - (-3)^{-2}$.

147. $8^{\frac{2}{3}} + 9^{\frac{3}{2}}$.

148. $8^{-\frac{1}{3}} - 9^{-\frac{1}{2}}$.

149. $a^2b \cdot a^{-1}b^2 \cdot ab^{-1}$.

150. $2\,a \cdot 3\,a^{\frac{1}{2}} \cdot a^{-\frac{2}{3}}$.

151. $3\,a^{\frac{1}{3}}x^2y \cdot abcx^{-1}$.

152. $a^2\sqrt{x} \cdot a^{\frac{1}{2}}x^{-1}$.

153. $2\sqrt{a} \cdot 3\,a^{\frac{1}{2}}\sqrt{x} \cdot x^{-\frac{1}{2}}$.

154. $a^{\frac{1}{2}}\sqrt{x} \cdot a^{\frac{1}{3}}\sqrt{x^{-1}} \div a\sqrt{x^3}$.

155. $2\,a^{-1}\sqrt{x^3} \cdot 3\,a^{\frac{1}{2}}\sqrt{x^{-1}} \cdot ax^{-1}$.

156. $x^{-\frac{3}{2}}a\sqrt{c^{-1}d} \div x^{-2}a^{\frac{1}{2}}\sqrt{cd^{-1}}$.

157. $\dfrac{a\sqrt{a}}{x\sqrt{x}}$.

158. $\dfrac{a^{-1}\sqrt{a}}{x^{-1}\sqrt{x}}$.

159. $\dfrac{a^{\frac{1}{2}}\sqrt{x^3}}{\sqrt{a^3x^5}}$.

160. $\dfrac{a^{\frac{1}{3}}\sqrt{a^2\sqrt{x}}}{a^2x}$.

161. $\dfrac{2\,x^{-2}\sqrt{x^{-3}}}{3\,x^{-\frac{7}{2}}}$.

162. $\dfrac{a^2c^{\frac{3}{4}}\sqrt{a^{-1}}}{\sqrt{ac\sqrt{ac}}}$.

163. $\dfrac{2\,a^{-\frac{1}{2}}\sqrt{m^3}}{3\,m^{-\frac{1}{3}}\sqrt{a^3}}$.

164. $\dfrac{ym^{-1}\sqrt{m^3}}{n^{-1}\sqrt{mn^2}}$.

165. $\dfrac{6\,a^2b^{\frac{1}{2}}\sqrt{4\,a^{-4}b^{-1}}}{3\,x^{-\frac{3}{2}}\sqrt{x^3}}$.

166. $\dfrac{a^{-1}\sqrt[3]{ax^{-1}}}{x^{-1}\sqrt[3]{a^{-1}x}}$.

167. $\dfrac{5\,a^{-1}\sqrt{x^{-3}}}{2\,a^{\frac{1}{2}}\sqrt{x^{-1}}}$.

168. $\dfrac{a^{\frac{1}{m}}\sqrt[m]{a^2c^2}}{c^{\frac{1}{m}}\sqrt[m]{ac^3}}$.

169. $\dfrac{x^{\frac{m}{3}}\sqrt[m]{a^{-3}}}{a^{\frac{1}{m}}\sqrt{x^{-m}}}$.

170. $\dfrac{2\,a\sqrt{9\,ax^{-1}}}{3\,c\sqrt[3]{27\,x^{-2}a^{-1}}}$.

171. $\dfrac{\sqrt[4]{m\sqrt[3]{m^{-2}}}}{\sqrt[12]{m^{-5}}}$.

172. $\dfrac{\sqrt{4\,a^6x^{-2}}}{3\,x^{\frac{1}{2}}\sqrt[3]{-64\,a^{-1}x}}$.

173. $\dfrac{\sqrt[3]{a\sqrt{a\sqrt{a}}}}{\sqrt[4]{\sqrt[3]{a^{-5}}}}$.

174. $\dfrac{\sqrt[3]{-27\sqrt{64\,a^{12}}}}{-\sqrt[3]{-125\,a^{-6}}}$.

Reduce to the simplest form:

175. $(a^2)^2$.

176. $(a^2)^3$.

177. $(a^2)^{-1}$.

178. $(a^3)^{-2}$.

179. $(a^{-1})^2$.

180. $(2\,a^{-1})^2$.

181. $(2\,a^{-3})^{-2}$.

182. $(a^{-2})^{-\frac{1}{2}}$.

183. $(4\,a^{-3})^{\frac{1}{2}}$.

184. $(a^{-1})^{-\frac{1}{3}}$.

185. $(3x^{\frac{1}{2}})^{-3}$.

186. $(x^{-\frac{1}{2}})^{-2}$.

187. $(5ab)^2$.

188. $(a^2b^2)^3$.

189. $(a^{-1}b^{-1})^2$.

190. $(ab^{-1})^{-2}$.

191. $(a^4b^4c^{-2})^{\frac{1}{2}}$.

192. $(64\,bc^{-2})^{-\frac{1}{3}}$.

193. $(-4a)^2$.

194. $(-ab)^3$.

195. $(-2a^2b)^4$.

196. $(-2ab^3)^3$.

197. $(-5a)^{-2}$.

198. $(-3a^n)^{-3}$.

199. $(2^{-1}a^{-\frac{1}{2}})^4$.

200. $(8^{-\frac{1}{3}})^2$.

201. $(-8^{-\frac{2}{3}})^2$.

202. $(-27^{-4})^{\frac{1}{3}}$.

203. $(-125)^{-\frac{1}{3}}$.

204. $(-8a^2)^{\frac{2}{3}}$.

205. $(2a^{-1}c^{\frac{1}{3}})^3$.

206. $(2a^{\frac{1}{2}}x^{-1}y)^{-2}$.

207. $(2m^2np^{-1})^{-3}$.

208. $(\sqrt[3]{-8a^{-1}})^2$.

209. $(\sqrt[4]{16a^4b^8})^{-3}$.

210. $(2a \div 3a^{-1})^{-2}$.

211. $(8a \div 2c^{-3})^{-\frac{3}{2}}$.

212. $(\sqrt[5]{-32c^{10}})^{-3}$.

213. $\sqrt[3]{-81^{-\frac{3}{4}}a^6}$.

214. $(9a^{-1}y^{-2})^{-\frac{3}{2}}$.

215. $(16a^{-1}x^2y^{\frac{1}{2}})^{-\frac{5}{4}}$.

216. $(a\sqrt{a^{-1}})^2$.

217. $(a^{-1}\sqrt{a^3})^2$.

218. $(a^{-1}\sqrt{a\sqrt{a^3}})^{-\frac{1}{2}}$.

219. $(a^{-\frac{3}{2}}\sqrt{a^{-2}})^{-\frac{2}{5}}$.

220. $(2a^{-\frac{2}{3}}\sqrt[3]{a^{-4}})^{-1}$.

221. $(ab^{-1}\sqrt{a^{-1}b})^{-\frac{1}{3}}$.

222. $(a\sqrt{4a^{-4}})^{-3}$.

223. $(x^{-1}\sqrt{x\sqrt{16}})^3$.

224. $\{(\sqrt{a^{-1}b})^2\}^3$.

225. $\{(\sqrt{8a^4b^6})^{\frac{2}{3}}\}^4$.

226. $[(\sqrt[3]{27^2a^{-1}})^{\frac{3}{2}}]^{-1}$.

227. $\sqrt{16^{-1}x\sqrt[3]{a^{-1}}}$.

228. $(\sqrt{(\frac{16}{81})^{-\frac{1}{2}}})^{\frac{2}{3}}$.

229. $\sqrt[3]{-8\sqrt{a^5x^5}}$.

230. $(\sqrt[3]{\sqrt[5]{64\,a^9}})^5$.

231. $(a^{-1}\sqrt[3]{27\,a^{-\frac{3}{4}}x^{\frac{1}{2}}})^{-4}$.

232. $[\sqrt[7]{(\sqrt[5]{-128})^{-3}}]^5$.

233. $(\sqrt[3]{125\,a^2\sqrt{c^{-3}}})^{-3}$.

234. $\left[(64\,a^3)^{-\frac{1}{3}} \cdot \frac{1}{27^{-1}c^{-3}}\right]^{-2}$.

235. $\sqrt{(16\,x^2y^{-3}\sqrt{xy})^{-3}}$.

236. $\left(\frac{m\sqrt[4]{x^3}}{\sqrt{n}\sqrt{m}} \div \frac{\sqrt{m^{-1}}}{\sqrt[3]{mx} \cdot m^{-\frac{1}{6}}}\right)$.

237. $(\sqrt[3]{27\,x^{-1}})^{-2} \div (\sqrt{9\,x^{-3}})^{-1}$.

238. $\left(m^{-\frac{1}{3}}n^{\frac{1}{3}}\sqrt{mn^{-\frac{1}{3}}\sqrt[4]{n^{\frac{4}{3}}}}\right)^{\frac{1}{3}}$.

239. $\left(\sqrt{\frac{4\,a^2\sqrt{x}}{25\,x^2\sqrt[3]{a}}}\right)^{-3}$.

240. $\sqrt{25\,a^{-1}b\sqrt{x^{-3}}} \div \sqrt{9\,a^{-5}b^{-1}\sqrt{x^5}}$.

241. $\left\{\frac{\sqrt[3]{a}}{\sqrt[4]{x^{-1}}} \cdot \left(\frac{\sqrt[4]{x}}{a^{\frac{1}{3}}}\right)^2 \div \frac{\sqrt[3]{8\,a^{-1}}}{\sqrt{9\,x^{-1}}}\right\}^{-6}$.

242. $\left(\frac{\sqrt[3]{m^2n^{-1}}}{\sqrt[6]{mn^3p}} : \sqrt{mn^2p^3}\right)^{-3}$.

243. $\left\{\sqrt[3]{-27\,a^8b^{-5}c} \cdot \sqrt{\frac{\sqrt[3]{a^{-1}bc^4}}{49\,a^3b\sqrt{c}}}\right\}^{-2}$.

244. $\left[\frac{\sqrt[3]{\sqrt{a^{-2}c^3}}}{\sqrt{a^3c^2m^{\frac{2}{3}}}} \cdot \sqrt[5]{-a^{10}c^{-5}m}\right]^{-\frac{1}{2}}$.

245. $\left\{\sqrt[4]{\frac{n^{-\frac{1}{3}}a^3}{81\,a^{-1}\sqrt{x}}} \cdot \sqrt[3]{\frac{8\,x^{\frac{1}{2}}\sqrt{a^3n}}{x^{\frac{1}{4}}\,a^{\frac{3}{4}}\sqrt[8]{n^5}}}\right\}^{-1}$.

246. $\sqrt{\left(\frac{c^{\frac{3}{5}}d^{-1}\sqrt{b}}{\sqrt[4]{b^2}} \cdot \frac{c^{-3}\sqrt[5]{cd^5}}{\sqrt{c^{-2}\sqrt{ad^{-2}}}}\right)^{-1}}$.

247. $\left\{\sqrt[5]{\frac{x^{-2}\sqrt{a}}{x^{\frac{1}{2}}\,a^{-2}}} \cdot \sqrt[3]{\frac{27\,a\sqrt{x}}{\sqrt{ax^{-2}}}}\right\}^{-4}$.

248. $\left[\sqrt[3]{x^{-2}y\sqrt{\frac{xz}{\sqrt{y}}}} \div \sqrt[4]{\frac{xy}{z}(\sqrt{z^{-1}})^2}\right]^2$.

249. $\left\{\sqrt{\frac{\sqrt[3]{a^{-2}b^3}}{25\,a^3b^{-1}\sqrt[5]{c^2}}} \cdot \sqrt[5]{-32\,a^{10}b^{-5}c}\right\}^{-3}$.

250. $\left(\sqrt{a^2b\left(\sqrt{\frac{a^{-1}c}{b}}\right)^{-1} \div \sqrt[3]{\frac{ab}{c} \cdot \frac{b^{-1}c}{a^{-4}}}}\right)^{-6}$.

251. $\left(\sqrt{\frac{4\,ay^{-3}}{\sqrt[7]{x^2}}}\right)^{-3}\left(\frac{y^{\frac{1}{3}\frac{4}{3}}x^{-1}}{a^{-\frac{2}{4}\frac{1}{4}}}\right)^{\frac{2}{3}}\left(\frac{y^4}{2\,x}\right)^{-1}$.

252. $\left(\sqrt{a^2b\ {}^{1}c\sqrt{b^2c\ {}^{-1}a\sqrt{c^2a^{-1}b\sqrt{abc}}}}\right)^{16}$

Collect:

253. $(\frac{1}{8})^{\frac{1}{3}} + (\frac{1}{16})^{\frac{1}{4}} + (32)^{\frac{3}{5}}$.

254. $(\frac{2}{3})^{-2} + 8^{\frac{5}{3}} - 4^{-\frac{1}{2}}$.

255. $3^{-2} - 27^{-\frac{2}{3}} + 9^{-\frac{3}{2}}$.

256. $\left(\frac{2}{5}\right)^{-3} + 16^{-\frac{3}{4}} + \frac{5}{2^{-2}} + (-2)^{-2}$.

257. $(\frac{3}{4})^{-1} - 2^{-3} + (1.6)^{\frac{1}{2}} + 128^{-\frac{3}{7}}$.

258. $9^{-\frac{3}{2}} + 13\,x^0 + 1^{-\frac{3}{5}} + (\frac{1}{2})^{-4}$.

259. $\sqrt[3]{2 \cdot 4^{-5}} - \frac{\sqrt[7]{2}}{2^{-\frac{5}{7}}} + \frac{1}{8^{-\frac{4}{3}}} - 128^{-\frac{3}{7}}$.

260. $7\,x^0 - (7\,x)^0 - 1^{7x} + \frac{3}{2^{-2}}$.

261. $81^{-\frac{3}{4}} - \frac{3}{27^{-\frac{2}{3}}} + \frac{\sqrt[4]{3^3}}{\sqrt{3^{-\frac{1}{2}}}} - \frac{\sqrt[5]{3}}{9^{-\frac{2}{5}}}$.

Simplify:

262. $2^n \cdot 2^2 \div 2^n$.

263. $4^{n-2} \cdot 8^{2-n} \cdot 2^n$.

264. $(4^{n-1} \cdot 2^{1-2n}) \div 8$.

265. $(x^{m-1})^{m+1} \cdot (x^{-m})^m \cdot x^2$.

266. $(ab)^{x+y} \div a^y b^y$.

267. $a^{m+n} \cdot a^{2m-n} \cdot a^{n-3m}$.

268. $a^{\frac{m}{m+n}} \div a^{\frac{n}{m+n}}$.

269. $[(a^{x+y})^{x-y} \div (a^{y-x})^x]$.

270. $(x^{m-n})^p (x^{p-n})^m (x^{p-m})^n$.

271. $(a^{x+1})^2 (a^{2-x})^3 \cdot (a^{x-3})^{-1}$.

272. $(a^{\frac{m}{4}} b^{-\frac{n}{3}} c^{mn})^{\frac{12}{mn}}$.

273. $(3^{n+2} + 3 \cdot 3^n) \div (9 \cdot 3^{n+2})$.

274. $(x^{2a-3b} \cdot x^{-5a-b}) \div x^{-5b-3a}$.

275. $\{[(a)^{m-1}]^{\frac{1}{m^2-1}}\}^{mn+n}$.

276. $[\{(x^a)^{-b}\}^{-c}] \div [\{(x^c)^{-a}\}^{-b}]$.

277. $(\sqrt[m]{a^{m^2}} \div \sqrt[n]{a})^{\frac{m}{mn-1}}$.

278. $\left[\left(\frac{x^{a+1}}{x^{1-a}}\right)^{a} \div \left(\frac{x^{a}}{x^{1-a}}\right)^{a-1}\right]^{\frac{1}{3a-1}}$.

279. $\left[\frac{(a^{mn})^{-3}}{a^{n}}\right]^{\frac{1}{3m+1}}$.

280. $\{(c^{m+1})^{\frac{1}{m}}\}^{-\frac{m^2}{m^2-1}} \cdot c^{\frac{1}{m+1}}$.

281. $\sqrt{\frac{2^{n+2}}{4^{-n}} \div \frac{8^{n}}{2^{3}}}$.

282. $\frac{2^{n+1}\sqrt{2 \cdot 2^{n-3}}}{2^{\frac{3n}{2}}\sqrt{2^{-3}}}$.

283. $([x^{\frac{a+b}{c}}] \cdot [x^{\frac{a-b}{c}}] \div x^{\frac{2a}{c}\frac{1}{c}})^{c}$.

284. $\left\{\sqrt[m+n]{\left(\frac{x^{m}}{x^{n}}\right)^{mn} \cdot \left[\frac{(x^{m+n})^{n}}{(x^{m-n})^{m}}\right]^{n}}\right\}^{\frac{1}{n^2}}$.

285. $\left[\sqrt[a]{\left(\frac{x^{m}}{x^{n}}\right)^{c}\left(x^{a}\right)\left(\frac{x^{n}}{x^{m}}\right)^{c}}\right]^{-a^2}$.

II. MISCELLANEOUS APPLICATIONS OF EXPONENTS

Exercise 46

Multiply:

1. $a^{-2} - 2a^{-1}b^{-1} + b^{-2}$ by $a^{-1} - b^{-1}$.
2. $a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}$ by $a^{\frac{1}{3}} + b^{\frac{1}{3}}$.
3. $a^{-2} - 2a^{-1} + 3 - a$ by $3a^{-1} - 2 - 2a$.
4. $a^{\frac{1}{2}} - a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}$ by $a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}$.
5. $a^{-3} - 2a^{-2} + 3a^{-1} + 1$ by $a^{-2} - 3a^{-1} - 1$.
6. $3a^{\frac{4}{3}} - 6a^{\frac{2}{3}} + 4$ by $a^{\frac{4}{3}} + 2a^{\frac{2}{3}} - 3$.
7. $3a^{\frac{2}{3}} - a^{\frac{1}{3}} + 2 - 4a^{-\frac{1}{3}}$ by $2a^{\frac{1}{3}} - 3 + 2a^{-\frac{1}{3}}$.
8. $\frac{2a^{\frac{3}{2}}}{\sqrt[4]{x^3}} + \frac{3a}{\sqrt{x}} - \frac{a^{\frac{1}{2}}}{\sqrt[4]{x}} + 2 - \frac{\sqrt[4]{x}}{\sqrt{a}}$ by $\frac{\sqrt{a}}{\sqrt[4]{x}} - 2 + \frac{\sqrt[4]{x}}{\sqrt{a}}$.

Multiply the following by inspection:

9. $(a^{-1}+1)^2$.
10. $(a^{-2}+3)^2$.
11. $(a^{-\frac{2}{3}}-4)^2$.
12. $(a^{-1}+b^{-1})^2$.
13. $(a^{-\frac{1}{2}}+b^{-1})^2$.
14. $(a^{-1}+b)(a^{-1}-b)$.
15. $(a^{-1}+3)(a^{-1}-3)$.
16. $(a^{-2}-4)(a^{-2}-1)$.
17. $(a^{\frac{1}{2}}-2)(a^{\frac{1}{2}}+2)$.
18. $(a^{-2}-3)(a^{-2}+2)$.
19. $(a^{\frac{1}{2}}+4)(a^{\frac{1}{2}}+5)$.
20. $(5-a^{-2})(3+a^{-2})$.
21. $(a^{\frac{1}{2}}-b^{\frac{1}{2}})(a^{\frac{1}{2}}+b^{\frac{1}{2}})$.
22. $(a^{-\frac{1}{2}}-3)(a^{-\frac{1}{2}}-2)$.
23. $(a^{-\frac{1}{3}}-a^{-1})(a^{-\frac{1}{3}}-2\,a^{-1})$.
24. $(a^{-1}b+c^{-1})(2\,a^{-1}b-c^{-1})$.
25. $(a^{-1}+b^{-1}+1)^2$.
26. $(a^{-2}+b^{-2}-c)^2$.

Divide:

27. $a^{-3}+3\,a^{-2}+3\,a^{-1}+1$ by $a^{-1}+1$.
28. $a^{-\frac{4}{3}}+2\,a^{-\frac{2}{3}}+1$ by $a^{-\frac{2}{3}}+1$.
29. $a+b$ by $a^{\frac{1}{3}}+b^{\frac{1}{3}}$.
30. $a-125\,c^{-1}$ by $a^{\frac{1}{3}}-5c^{-\frac{1}{3}}$.
31. $a^{\frac{2}{4}}-6\,a^{\frac{1}{2}}+12\,a^{\frac{1}{4}}-8$ by $a^{\frac{1}{4}}-2$.
32. $x^{\frac{1}{3}}+a^{-\frac{1}{3}}$ by $x^{\frac{1}{3}}+a^{-\frac{1}{3}}$.
33. $a^{\frac{3}{4}}-3\,a^{\frac{1}{4}}+3\,a^{-\frac{1}{4}}-a^{-\frac{3}{4}}$ by $a^{\frac{1}{4}}-a^{-\frac{1}{4}}$.
34. $10\,x^{-4}-27\,x^{-3}+34\,x^{-2}-18\,x^{-1}-8$ by $5\,x^{-2}-6\,x^{-1}-2$.
35. $12\,a^{-\frac{6}{5}}-17\,a^{-\frac{3}{5}}-9+13\,a^{\frac{3}{5}}-63\,a^{\frac{6}{5}}$ by $4\,a^{-\frac{3}{5}}-3+7\,a^{\frac{3}{5}}$.
36. $6\,a^{-1}+11\,a^{-\frac{3}{4}}\sqrt[3]{x}-\dfrac{26\,x^{\frac{2}{3}}}{\sqrt{a}}-\dfrac{21\,x}{\sqrt[4]{a}}+10\,x^{\frac{4}{3}}$ by $\dfrac{2}{\sqrt{a}}+5\,a^{-\frac{1}{4}}x^{\frac{1}{3}}-2\,x^{\frac{2}{3}}$.

Divide the following by inspection:

37. $(a^{-2}-9)$ by $(a^{-1}+3)$.
38. $(a^{-2}-b^{-2})$ by $(a^{-1}-b^{-1})$.
39. $(a^{-4}-16)$ by $(a^{-2}+4)$.
40. $(a-81)$ by $(a^{\frac{1}{2}}-9)$.
41. $(a^{-3}-b^{-3})$ by $(a^{-1}-b^{-1})$.
42. $(a-8)$ by $(a^{\frac{1}{3}}-2)$.
43. $(a^{-\frac{3}{2}}+27)$ by $(a^{-\frac{1}{2}}+3)$.
44. $(a^{-1}-64)$ by $(a^{-\frac{1}{3}}-4)$.
45. $(a^{-4}-b^{-4})$ by $(a^{-1}-b^{-1})$.
46. $(a^{-4}-16)$ by $(a^{-1}-2)$.

Factor:

47. $a^{-2}-b^{-2}$.
48. $a^{-4}-81$.
49. $4a^{-2}-25b^{-2}$.
50. $a^{\frac{1}{4}}-9$.
51. $a^{\frac{3}{4}}-8$.
52. $a^{-2}+5a^{-1}+6$.
53. $a^{-2}-6a^{-1}y^{-1}+5y^{-2}$.
54. $a^{\frac{2}{3}}-10a^{\frac{1}{3}}+25$.
55. $a^{\frac{1}{2}}+8a^{\frac{1}{4}}+16$.
56. $m^{-\frac{4}{3}}-5m^{-\frac{2}{3}}-36$.
57. $x^{-4}-9x^{-2}+8$.
58. $x^{-3}-8x^{-2}-48x^{-1}$.
59. $2x^{-2}+x^{-1}-10$.
60. $4x^{-\frac{2}{3}}+11x^{-\frac{1}{3}}-3$.
61. $x^{\frac{3}{2}}-27$.
62. $x^{\frac{3}{5}}+64$.
63. $x^{\frac{6}{7}}-8$.
64. $x-4$.
65. $8a^{3}+b^{3}$.
66. $a^{-1}+125b^{-9}$.

Simplify:

67. $a^{-1}+b^{-1}$.
68. $a^{-2}+b^{-2}$.
69. $a^{-1}b+ab^{-1}$.
70. $a^{-1}bc+ab^{-1}c+abc^{-1}$.
71. $a^{-1}(a+b)+(a+b)b^{-1}$.
72. $(x^{-1}+y^{-1})\div(x^{-1}-y^{-1})$.
73. $(1+mn^{-1})\div(1+m^{-1}n)$.
74. $\dfrac{(a+1)(a-1)^{-1}+1}{(a+1)(a-1)^{-1}-1}$.
75. $\dfrac{1-(c+1)^{-1}}{1+(c-1)^{-1}}$.
76. $\dfrac{x+5+6x^{-1}}{1+6x^{-1}+8x^{-2}}$.
77. $\dfrac{a(a-1)^{-1}+b(a+1)^{-1}}{a(a+1)^{-1}+b(a-1)^{-1}}$.

78. $\dfrac{mn^{-2}+m^{-2}n}{m^{-2}-m^{-1}n^{-1}+n^{-2}}$.

79. $\dfrac{x(1+x)^{-1}+x^{-1}(1-x)}{x(1+x)^{-1}-x^{-1}(1-x)}$.

80. $\left[\dfrac{a^{-1}+(m+n)^{-1}}{a^{-1}-(m+n)^{-1}}\right][1+(m^2+n^2-a^2)2^{-1}m^{-1}n^{-1}]$.

81. $[(m+a)^{-1}(m-c)^{-1}+(m-a)^{-1}(m+c)^{-1}]$
$\div[(m+a)^{-1}(m+c)^{-1}+(m-a)^{-1}(m-c)^{-1}]$.

Expand:

82. $(x-2x^{-2})^3$.

83. $(2x^{-1}+3)^5$.

84. $(x^{-1}-3x^2)^4$.

85. $(x+2x^{-2})^3$.

86. $(\sqrt{x}-\sqrt[3]{x})^3$.

87. $(\sqrt{x}-3\sqrt{x^5})^4$.

88. $\left(2\sqrt{x^{-1}}-\dfrac{3}{x^2}\right)^4$.

89. $\left(2a^{-\frac{1}{2}}-\dfrac{a^{\frac{1}{2}}}{\sqrt{x}}\right)^5$.

Extract the square root of:

90. $x^{-\frac{8}{3}}-10x^{-\frac{8}{4}}+25$.

91. $x-8x^{\frac{3}{4}}+18x^{\frac{1}{2}}-8x^{\frac{1}{4}}+1$.

92. $9a^{-2}-6a^{-1}+13-4a+4a^2$.

93. $9x^{-4}-30x^{-3}+67x^{-2}-70x^{-1}+49$.

94. $4x^{\frac{2}{3}}-4x^{\frac{1}{2}}+13x^{\frac{1}{3}}-6x^{\frac{1}{6}}+9$.

95. $9x-12x^{\frac{1}{4}}+34x^{-\frac{1}{2}}-20x^{-\frac{5}{4}}+25x^{-2}$.

96. $16x^{-\frac{2}{3}}-\dfrac{24}{\sqrt[3]{x}}-7+12\sqrt[3]{x}+4\sqrt[3]{x^2}$.

97. $\dfrac{9a}{b}-\dfrac{24\sqrt{a}}{\sqrt{b}}-\dfrac{24\sqrt{b}}{\sqrt{a}}+\dfrac{9b}{a}+34$.

Solve the following equations:

98. $x^{-1}=2$.
99. $x^{\frac{1}{3}}=3$.
100. $x^{\frac{1}{3}}=-2$.
101. $x^{-\frac{1}{2}}=2$.
102. $x^{-\frac{1}{4}}=-3$.
103. $x^{\frac{2}{3}}=8$.
104. $x^{\frac{3}{2}}=-27$.
105. $x^{-\frac{3}{2}}=-8$.
106. $x^{\frac{1}{3}}=\frac{1}{2}$.
107. $x^{-\frac{1}{3}}=-\frac{1}{2}$.
108. $x^{-\frac{2}{3}}=\frac{1}{4}$.
109. $x^{-\frac{4}{3}}=16$.
110. $x^{n}=2$.
111. $x^{n}=2^{m}$.
112. $x^{-\frac{1}{n}}=-2$.
113. $x^{-\frac{3}{4}}=\frac{1}{64}$.
114. $x^{-\frac{3}{4}}=\frac{1}{125}$.
115. $x^{-\frac{1}{n}}=1$.
116. $\sqrt{x^{-3}}=1000$.
117. $x^{\frac{2}{3}}=\sqrt[3]{5}$.
118. $x^{\frac{3}{4}}=\sqrt{\frac{27}{125}}$.
119. $(x+1)^{2}=4$.
120. $(x+2)^{3}=125$.
121. $(x-1)^{\frac{1}{2}}=3$.
122. $(x-5)^{\frac{1}{2}}=1$.
123. $(x+1)^{\frac{2}{3}}=4$.
124. $(x-3)^{\frac{3}{2}}=8$.
125. $(x+4)^{-3}=27$.
126. $(3x-1)^{-2}=\frac{1}{9}$.
127. $(x^{2}-1)^{-2}=\frac{1}{9}$.
128. $(2x-1)^{-2}=\frac{1}{25}$.
129. $(x^{-1}+1)^{-2}=9$.
130. $(x^{-\frac{1}{2}}+2)^{-2}=16$.
131. $(x^{-\frac{2}{3}}-5)^{-\frac{1}{2}}=\frac{1}{2}$.
132. $(x^{-\frac{1}{2}}-\frac{1}{2})^{-\frac{1}{2}}=\frac{1}{2}$.
133. $(x^{-1}+3)^{-n}=1$.
134. $(x^{-\frac{6}{2}}-7)^{a}=1$.

Find the value of x in the following:

135. $4^{x}=8$.
136. $4^{x}=64$.
137. $16^{x}=8$.
138. $9^{x}=27$.
139. $9^{x}=\frac{1}{27}$.
140. $27^{x}=3$.
141. $8^{x}=\frac{1}{32}$.
142. $3^{2x}=9$.
143. $(\frac{1}{2})^{-x}=8$.

Find the value of x in the following:

144. $x^{-1}=y$; $y^{2}=4$.
145. $x^{-1}=y$; $y^{\frac{1}{2}}=2$.
146. $x^{-3}=y^{-2}$; $y^{-1}=-8$.
147. $x^{\frac{1}{2}}=y^{\frac{1}{3}}$; $y^{\frac{1}{2}}=2$.
148. $x^{\frac{1}{2}}=y^{\frac{1}{3}}$; $y^{\frac{2}{3}}=4$.
149. $x^{-\frac{2}{3}}=y^{\frac{1}{2}}$; $y^{-1}=-2$.
150. $x^{-\frac{3}{2}}=y^{-1}$; $y^{\frac{2}{3}}=4$.
151. $x^{-\frac{1}{2}}=y^{-\frac{4}{3}}$; $y^{-\frac{2}{3}}=4$.

Find the value of n in the following:

152. $2^{n-1}=16$.
153. $3^{n-1}=27$.
154. $9^{n-1}=27$.
155. $4^{n+1}=16$.
156. $16^{n-1}=8$.
157. $3^{n+1}=\frac{1}{27}$.
158. $4^{n-2}=\frac{1}{16}$.
159. $(\frac{1}{2})^{n-1}=\frac{1}{64}$.
160. $(\frac{1}{4})^{n-1}=\frac{1}{32}$.
161. $(\frac{2}{3})^{n-2}=\frac{8}{27}$.

RADICALS

I. TRANSFORMATIONS

Exercise 47

Reduce to the simplest form:

1. $\sqrt{8}$.
2. $\sqrt{12}$.
3. $\sqrt{20}$.
4. $\sqrt{28}$.
5. $\sqrt{27}$.
6. $\sqrt{45}$.
7. $\sqrt{48}$.
8. $\sqrt{72}$.
9. $-\sqrt{125}$.
10. $\sqrt[3]{16}$.
11. $\sqrt[3]{24}$.
12. $-\sqrt[3]{54}$.
13. $\sqrt[3]{108}$.
14. $-\sqrt[3]{128}$.
15. $\sqrt[3]{-81}$.
16. $\sqrt[4]{32}$.
17. $\sqrt[4]{162}$.
18. $-\sqrt[5]{96}$.
19. $\sqrt{a^3}$.
20. $\sqrt{a^2b}$.
21. $\sqrt{a^3b^2}$.
22. $\sqrt[3]{a^4b^3c^2}$.
23. $-\sqrt{18\,a^3}$.
24. $\sqrt{27\,a^5}$.
25. $\sqrt[3]{54\,a^4b^5}$.
26. $-\sqrt[3]{128\,m^4n^3}$.
27. $-\sqrt[3]{320\,a^3b^5c^7}$.
28. $3\sqrt{27\,a}$.
29. $4\sqrt{28\,a^3b^3}$.
30. $2\sqrt{56\,m^4n^3}$.
31. $\sqrt{20\,m^{14}}$.
32. $-2\sqrt[3]{250\,a^4}$.
33. $\sqrt[3]{16\,x^{10}y^{16}}$.
34. $3\sqrt[4]{64\,m^5n}$.
35. $\frac{1}{4}\sqrt{54\,a^4}$.
36. $\frac{1}{2}\sqrt{4\,x^{15}}$.
37. $-\frac{1}{3}\sqrt{20\,bc^3d}$.
38. $\sqrt[3]{54\,a^{18}b^{20}}$.
39. $-\frac{1}{5}\sqrt{125\,c^{17}}$.

40. $\sqrt[3]{27\,a^{27}x^{64}}$.

41. $a\sqrt{(a+c)^3}$.

42. $-a\sqrt{3\,a^2(a+1)^2}$.

43. $5\,m\sqrt[3]{(a-1)^4}$.

44. $-a\sqrt[4]{a^5(a-1)^4}$.

45. $\sqrt{a^3+2\,a^2x+ax^2}$.

46. $\sqrt{36(a^2-x^2)(a+x)}$.

47. $\sqrt{\frac{128\,a^4x}{9\,c^2}}$.

48. $5\sqrt{\frac{32\,m^4n^3}{25\,x^2}}$.

49. $m^2\sqrt{\frac{3\,a}{m^4n^2}}$.

50. $\frac{7}{18}\sqrt{\frac{108\,a^9x^7}{49\,c^4}}$.

Change to entire surds:

51. $2\sqrt{5}$.

52. $3\sqrt{7}$.

53. $4\sqrt{3}$.

54. $2\sqrt[3]{5}$.

55. $3\sqrt[3]{4}$.

56. $2\sqrt[4]{7}$.

57. $2\,a\sqrt{a}$.

58. $3\,a\sqrt[3]{a}$.

59. $-2\,a\sqrt[3]{a}$.

60. $3\,a^2\sqrt[3]{a^2}$.

61. $2\,a\sqrt[4]{3\,a^3}$.

62. $-2\,a\sqrt[3]{5\,a}$.

63. $-a^2\sqrt[3]{2\,a^2}$.

64. $\frac{a}{2}\sqrt{a}$.

65. $\frac{3}{m}\sqrt{2\,a}$.

66. $\frac{x}{3\,a}\sqrt{2\,x}$.

67. $\frac{2\,a}{3}\sqrt[3]{\frac{3}{2\,a}}$.

68. $-\frac{a}{2}\sqrt[3]{a^2}$.

69. $-\frac{m}{3}\sqrt[3]{9}$.

70. $\frac{a+b}{2}\sqrt{\frac{1}{a+b}}$.

71. $\frac{a+b}{2}\sqrt[3]{\frac{1}{a+b}}$.

72. $-(a-1)\sqrt{\frac{1}{a^2-1}}$.

Reduce:

73. $\sqrt{\frac{1}{2}}$.

74. $\sqrt{\frac{1}{3}}$.

75. $\sqrt{\frac{1}{5}}$.

76. $\sqrt{\frac{1}{11}}$.

77. $\sqrt{\frac{2}{3}}$.

78. $\sqrt{\frac{3}{5}}$.

79. $\sqrt{\frac{5}{8}}$.

80. $\sqrt{\frac{4}{5}}$.

81. $\sqrt{\frac{3}{8}}$.

82. $\sqrt{\frac{1}{x}}$.

83. $\sqrt{\frac{a}{b}}$.

84. $\sqrt[3]{\frac{1}{2}}$.

85. $-\sqrt[3]{\frac{1}{3}}$.

86. $\sqrt[3]{\frac{x}{2}}$.

87. $-\sqrt[3]{\frac{x^2}{a}}$.

88. $\sqrt[3]{\frac{x^2}{3\,a}}$.

89. $\sqrt[3]{\frac{m}{2\,n}}$.

90. $2\sqrt{\frac{3\,x}{2}}$.

91. $-3\sqrt[3]{\frac{27}{4}}$.

92. $\frac{1}{2}\sqrt[4]{\frac{4\,ax}{27}}$.

93. $-\frac{a}{3}\sqrt{\frac{a^2x^2}{4}}$.

94. $-\frac{2}{3\,a}\sqrt[4]{\frac{a^3}{8}}$.

95. $-\frac{9}{2\,a}\sqrt{\frac{4\,ac}{3}}$.

96. $\frac{2\,c}{3\,a}\sqrt[3]{\frac{27\,a^4}{c^2}}$.

97. $\frac{1}{2}\sqrt{\frac{20}{7}}$.

98. $15\sqrt{\frac{7\,a^3}{90}}$.

99. $8\sqrt{\frac{5\,a^5}{12\,a^2}}$.

100. $\sqrt[3]{6\frac{2}{3}(m-n)^4}$.

101. $\sqrt[4]{2\frac{56}{27}(a+1)^5}$.

102. $\sqrt[3]{3\frac{43}{45}\,m^5n^7c^{10}}$.

Simplify the indices of:

103. $\sqrt[4]{m^2}$.

104. $\sqrt[3]{c^6}$.

105. $\sqrt[4]{9}$.

106. $\sqrt[6]{27}$.

107. $\sqrt[4]{25\,a^2b^2}$.

108. $\sqrt[3]{c^6d^{12}x^{15}}$.

109. $\sqrt[4]{9\,a^8c^{12}}$.

110. $\sqrt[8]{81\,a^4c^8d^{12}}$.

111. $\sqrt[2n]{16\,a^4b^2c^6}$.

112. $\sqrt[6]{27a^6c^{12}}$.

113. $\sqrt[4]{100}$.

Change to radicals having the same index:

114. $\sqrt{3}$ and $\sqrt[3]{5}$.

115. $\sqrt{5}$ and $\sqrt[3]{10}$.

116. $\sqrt[3]{9}$ and $\sqrt[4]{4}$.

117. $\sqrt{15}$ and $\sqrt[3]{50}$.

118. $\sqrt{11}$ and $\sqrt[3]{30}$.

119. $\sqrt[3]{25}$ and $\sqrt[4]{75}$.

120. $\sqrt{6}$, $\sqrt[3]{15}$, and $\sqrt[4]{35}$.

121. $\sqrt{3}$, $\sqrt[3]{7}$, and $\sqrt[6]{10}$.

122. $\sqrt{m}$, $\sqrt[4]{m^3}$, and $\sqrt[5]{m^2}$.

Which is the greater:

123. $2\sqrt{3}$ or $3\sqrt{2}$?

124. $\sqrt{11}$ or $\sqrt[3]{30}$?

125. $2\sqrt{3}$ or $\sqrt[3]{42}$?

126. $3\sqrt[3]{5}$ or $3\sqrt{3}$?

127. $2\sqrt[3]{4}$ or $\sqrt{10}$?

128. $\sqrt{\frac{5}{6}}$ or $\sqrt[3]{\frac{7}{9}}$?

Which is the greatest:

129. $\sqrt{5}$, $\sqrt[3]{10}$, or $\sqrt[4]{18}$?

130. $\sqrt{6}$, $\sqrt[3]{15}$, or $\sqrt[4]{35}$?

Collect:

131. $\sqrt{50}+\sqrt{18}-\sqrt{8}-\sqrt{32}$.

132. $\sqrt{18}-\sqrt{98}+\sqrt{50}-\sqrt{72}$.

133. $\sqrt{27}-\sqrt{12}+\sqrt{75}+\sqrt{3}$.

134. $\sqrt{12\,a}-\sqrt{27\,a}-\sqrt{48\,a}+\sqrt{108\,a}$.

135. $\sqrt{am^3}-\sqrt{a^3m}+\sqrt{9\,am^3}+\sqrt{4\,a^3m}$.

136. $\frac{1}{2}\sqrt{12}-\sqrt{50}+\frac{1}{3}\sqrt{48}-\sqrt{18}$.

137. $\sqrt{20}-\sqrt{\frac{1}{2}}+\sqrt{\frac{5}{9}}+4\sqrt{2}-3\sqrt{5}$.

138. $\sqrt{12}+\sqrt{\frac{1}{2}}-\sqrt{27}+\sqrt{\frac{1}{8}}-\sqrt{108}$.

139. $\sqrt{50}-\sqrt[4]{4}-6\sqrt{\frac{1}{2}}+3\sqrt{\frac{1}{3}}$.

140. $\sqrt[4]{\frac{1}{4}}-\sqrt{\frac{1}{8}}+\sqrt{98}-2\sqrt{27}$.

141. $2\sqrt[4]{\frac{1}{3}}+3\sqrt[6]{27}-2\sqrt[4]{144}$.

142. $2\sqrt{\frac{2}{5}}-3\sqrt{\frac{1}{10}}-\sqrt{\frac{5}{2}}+\sqrt{1000}$.

143. $3\sqrt{ax^3}+4\sqrt{\frac{ax^5}{25}}-\sqrt[4]{\frac{a^2x^6}{81}}$.

144. $\sqrt{50}-\sqrt[3]{432}+\sqrt{32}+\sqrt[3]{250}$.

145. $3\sqrt[3]{2}+\sqrt[3]{24}-\sqrt[3]{3}+\sqrt[3]{16}$.

146. $30\sqrt{\frac{1}{2}}-\frac{9}{2}\sqrt{8}+9\sqrt{84\frac{1}{2}}$.

147. $\sqrt{180}-2\sqrt{5}+15\sqrt{\frac{4}{5}}$.

148. $\frac{2}{3}\sqrt{162}+10\sqrt{4\frac{1}{2}}-13\sqrt{2}$.

149. $\sqrt[4]{36}-\sqrt{\frac{2}{3}}-4\sqrt{6}+2\sqrt{\frac{1}{6}}$.

150. $10\sqrt{12\frac{1}{2}}+7\sqrt{2}-3\sqrt{338}+5\sqrt[4]{4}+4\sqrt{\frac{9}{2}}$.

151. $\sqrt{24}-6\sqrt{\frac{1}{6}}+\frac{1}{2}\sqrt{96}-\sqrt{66\frac{2}{3}}+\frac{2}{5}\sqrt{\frac{2.5}{6}}$.

152. $6\sqrt{33\frac{1}{3}} - \sqrt{96} + \sqrt{\frac{2}{3}} - \frac{4}{3}\sqrt{\frac{27}{8}} + 4\sqrt{6}$.

153. $12\sqrt{16\frac{1}{3}} + 5\sqrt{3} - 5\sqrt{432} + 6\sqrt{\frac{100}{3}}$.

Multiply:

154. $(\sqrt{20} + \sqrt{80} + \sqrt{45})$ by $\sqrt{5}$.

155. $(\sqrt{8} - 2\sqrt{12} + \sqrt{20})$ by $\sqrt{6}$.

156. $(\sqrt[3]{4} - \frac{1}{2}\sqrt[3]{32} + \sqrt[3]{5})$ by $\sqrt[3]{16}$.

157. $(2\sqrt{3} - 2)(2\sqrt{3} + 2)$.

158. $(2\sqrt{5} + 3\sqrt{2})(3\sqrt{5} - 4\sqrt{2})$.

159. $(5\sqrt{3} - 2\sqrt{2})(3\sqrt{3} + 4\sqrt{2})$.

160. $(\sqrt{2} + \sqrt{3})^2$.

161. $(\sqrt{3} - 2\sqrt{2})^2$.

162. $(2\sqrt{3} - 2\sqrt{2})^2$.

163. $(3\sqrt{2} - 5\sqrt{3})^2$.

164. $(\sqrt{3} - \sqrt{2})^3$.

165. $(2\sqrt{2} - 2)^3$.

166. $(3\sqrt{2} - 2\sqrt{3})^3$.

167. $(\sqrt{7} - \sqrt{2} + \sqrt{5})(\sqrt{7} + \sqrt{2} - \sqrt{5})$.

168. $(3\sqrt{\frac{2}{3}} + 3\sqrt{\frac{1}{3}} - 10\sqrt{\frac{1}{5}})(\frac{1}{2}\sqrt{24} + \frac{1}{5}\sqrt{75} + \sqrt{20})$.

169. $(\sqrt{10 + \sqrt{19}})(\sqrt{10 - \sqrt{19}})$.

170. $(\sqrt{13 - 2\sqrt{22}})(\sqrt{13 + 2\sqrt{22}})$.

171. $(\sqrt{2 + x} + \sqrt{x})(\sqrt{2 + x})$.

172. $(\sqrt{a + 1} - 2)^2$.

173. $(2\sqrt{a^2 - 1} - 3)^2$.

174. $(\sqrt{m + 1} - \sqrt{m - 1})(\sqrt{m - 1})$.

175. $(\sqrt{m + 2} + \sqrt{m})(\sqrt{m + 2} - 2\sqrt{m})$.

176. $(m^2+m\sqrt{3}+3)(m\sqrt{3}-3)$.

177. $(\sqrt{m}-\sqrt{m-n}+\sqrt{n})(\sqrt{m}+\sqrt{m-n}+\sqrt{n})$.

178. $(\sqrt{a-1}+\sqrt{a+1})^2$.

179. $(\sqrt{21-6\sqrt{3}})(\sqrt{21+6\sqrt{3}})$.

180. $(5\sqrt{c^2+d^2}-4\sqrt{c^2-d^2})^2$.

181. $(2\sqrt{x}+\sqrt{4-x})(2\sqrt{x}-\sqrt{4-x})$.

182. $(\sqrt{a+1}-\sqrt{2})(2\sqrt{a+1}+\sqrt{2})(2a+\sqrt{2a+2})$.

183. $\sqrt{\frac{6}{35}}\cdot\sqrt[3]{\frac{175}{18}}$.

184. $\sqrt{\frac{5}{33}}\cdot\sqrt[4]{\frac{88}{25}}$.

185. $\sqrt{\frac{12}{35}}\cdot\sqrt[3]{\frac{245}{36}}$.

186. $(\sqrt{2}-2\sqrt[3]{4})(2\sqrt{2}-\sqrt[3]{4})$.

Divide:

187. $2\sqrt{32}$ by $3\sqrt{120}$.

188. $\sqrt[3]{81}$ by $\sqrt{3}$.

189. $\sqrt[3]{12}$ by $4\sqrt{2}$.

190. $\sqrt[4]{64}$ by $\sqrt[3]{32}$.

191. $\sqrt[3]{9a^2m}$ by $\sqrt[4]{3am^2}$.

192. $(\sqrt[3]{12}+4\sqrt{18})$ by $6\sqrt{2}$.

193. $\frac{1}{3}\sqrt{\frac{1}{135}}$ by $\frac{1}{2}\sqrt{\frac{1}{27}}$.

194. $(5\sqrt{18}-3\sqrt{27})$ by $3\sqrt{5}$.

195. $(2\sqrt{54}+\frac{1}{3}\sqrt{24})$ by $3\sqrt{\frac{1}{2}}$.

196. $\frac{3}{5}\sqrt{\frac{a}{c}}$ by $\frac{3}{10}\sqrt[3]{\frac{c}{a}}$.

197. $\left(\frac{2}{3}\sqrt{\frac{a}{c}}-\frac{1}{2}\sqrt{\frac{c}{a}}\right)$ by $\sqrt{ac}$.

198. $(12\sqrt{5}-8\sqrt{15}+3\sqrt{30})$ by $6\sqrt{10}$.

199. $10\sqrt{3}-15\sqrt{42}-9\sqrt{2}$ by $5\sqrt{6}$.

200. $(5\sqrt[3]{20}+3\sqrt[3]{45}+6\sqrt[3]{30})$ by $2\sqrt[3]{18}$.

201. $(\sqrt[4]{32}-\sqrt[4]{48}-\sqrt[4]{80})$ by $\sqrt[4]{3}$

202. $\sqrt{\frac{75a^3}{(m+n)^7}}$ by $\left(\frac{5a^2}{m+n}\sqrt{\frac{3a}{m+n}}\right)$.

II. MISCELLANEOUS APPLICATIONS OF RADICALS

Exercise 48

Extract the square root of:

1. $6+4\sqrt{2}$.
2. $11+6\sqrt{2}$.
3. $28-10\sqrt{3}$.
4. $21+8\sqrt{5}$.
5. $45-20\sqrt{5}$.
6. $42+12\sqrt{6}$.
7. $44-16\sqrt{7}$.
8. $30-12\sqrt{6}$.
9. $88-16\sqrt{10}$.
10. $57+12\sqrt{15}$.
11. $207-40\sqrt{14}$.
12. $82+12\sqrt{42}$.
13. $16+2\sqrt{39}$.
14. $74-6\sqrt{77}$.
15. $77-28\sqrt{7}$.
16. $a^2+b+2a\sqrt{b}$.
17. $a^2+2c+2a\sqrt{2c}$.
18. $m^4+m+2m^2\sqrt{m}$.
19. $2x+2\sqrt{x^2-1}$.
20. $m^2+(2m-2)\sqrt{2m-1}$.

Rationalize the denominators of:

21. $\dfrac{4}{\sqrt{2}}$.
22. $\dfrac{6}{\sqrt{3}}$.
23. $\dfrac{8}{\sqrt{2}}$.
24. $\dfrac{12}{\sqrt{6}}$.
25. $\dfrac{\sqrt{24}}{\sqrt{3}}$.
26. $\dfrac{3\sqrt{6}}{\sqrt{12}}$.
27. $\dfrac{2}{2+\sqrt{2}}$.
28. $\dfrac{3}{3-\sqrt{3}}$.
29. $\dfrac{5}{\sqrt{3}-2}$.
30. $\dfrac{2}{\sqrt{2}+\sqrt{3}}$.
31. $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$.
32. $\dfrac{3\sqrt{2}+2}{2\sqrt{2}-1}$.
33. $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.
34. $\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$.
35. $\dfrac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}$.
36. $\dfrac{3\sqrt{5}-2\sqrt{6}}{2\sqrt{5}-2\sqrt{6}}$.
37. $\dfrac{2\sqrt{a}+3\sqrt{c}}{2\sqrt{a}-3\sqrt{c}}$.
38. $\dfrac{5\sqrt{2x}-\sqrt{6x}}{5\sqrt{2x}+\sqrt{6x}}$.
39. $\dfrac{4\sqrt{2x}-3\sqrt{x}}{3\sqrt{2x}+5\sqrt{x}}$.
40. $\dfrac{a\sqrt{c}-b\sqrt{x}}{a\sqrt{c}+b\sqrt{x}}$.
41. $\dfrac{a+\sqrt{a^2-1}}{a-\sqrt{a^2-1}}$.
42. $\dfrac{\sqrt{x-3}+\sqrt{x}}{\sqrt{x-3}-\sqrt{x}}$.
43. $\dfrac{\sqrt{3a+1}+\sqrt{2a-1}}{\sqrt{3a+1}-\sqrt{2a-1}}$.
44. $\dfrac{\sqrt{m^2-2}-\sqrt{m^2+2}}{\sqrt{m^2-2}+\sqrt{m^2+2}}$.

Find, to three decimals, the numerical value of:

45. $\dfrac{1}{\sqrt{3}}$.

46. $\dfrac{8}{\sqrt{5}}$.

47. $\dfrac{3}{2\sqrt{3}}$.

48. $\dfrac{4}{\sqrt[3]{2}}$.

49. $\dfrac{3}{2\sqrt[3]{2}}$.

50. $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$.

51. $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$.

52. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$.

53. $\dfrac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-2\sqrt{2}}$.

54. $\dfrac{5\sqrt{3}+2\sqrt{2}}{\sqrt{2}-\sqrt{3}}$.

Simplify:

55. $\sqrt{15-6\sqrt{6}}$.

56. $\dfrac{\sqrt{8}}{(\sqrt{6}-\sqrt{2})(\sqrt{3}+1)}$.

57. $\left(a-1-\dfrac{\sqrt{3}}{\sqrt{2}}\right)\left(a-1+\dfrac{\sqrt{3}}{\sqrt{2}}\right)$.

58. $\dfrac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$.

59. $\dfrac{1}{a-\sqrt{a^2-4}}+\dfrac{1}{a+\sqrt{a^2-4}}$.

60. $\dfrac{1}{(3-\sqrt{2})^2}+\dfrac{1}{(3+\sqrt{2})^2}$.

61. $\dfrac{2\sqrt{x-1}}{3\sqrt{x+1}-2\sqrt{x-1}}+\dfrac{3\sqrt{x-1}}{3\sqrt{x+1}+2\sqrt{x-1}}$.

62. $\dfrac{\sqrt{26+8\sqrt{3}}}{\sqrt{6}-\sqrt{2}}$.

63. $\dfrac{\sqrt{19-8\sqrt{3}}}{\sqrt{3}+4}$.

64. $\dfrac{\sqrt{a}+\sqrt{b}}{2\sqrt{b}}-\dfrac{\sqrt{a}-\sqrt{b}}{2\sqrt{a}}$.

65. $\dfrac{\sqrt{3+\sqrt{2}}-\sqrt{2}}{\sqrt{3+\sqrt{2}}+\sqrt{2}}$.

66. $\dfrac{x+5}{\sqrt{x}+\sqrt{5}}-\dfrac{x-5}{\sqrt{x}-\sqrt{5}}$.

67. $\dfrac{(\sqrt{11+6\sqrt{2}})+(\sqrt{6+4\sqrt{2}})}{5-2\sqrt{2}}$.

68. $\left(\sqrt{\dfrac{a}{x}+\dfrac{x}{a}}-\sqrt{\dfrac{a}{x}-\dfrac{x}{a}}\right)^2$.

69. $\left(\sqrt{\dfrac{1+\sqrt{1-x^2}}{3}}+\sqrt{\dfrac{1-\sqrt{1-x^2}}{3}}\right)^2$.

70. $[3\sqrt{3}+(\sqrt{28-16\sqrt{3}})]^2$.

71. $\dfrac{(x+\sqrt{4-x^2})\,(x-\sqrt{4-x^2})}{(x+\sqrt{4-x^2})^2+(x-\sqrt{4-x^2})^2}$.

72. $\dfrac{\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}}$.

73. $\left(\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\dfrac{\sqrt{x+1}}{\sqrt{x-1}}-1}\right)\left(\dfrac{1}{\sqrt{x-1}}\right)$.

74. $\dfrac{3+\sqrt{2}}{\sqrt{3}+\sqrt{3+\sqrt{2}}}-\dfrac{3-\sqrt{2}}{\sqrt{3}-\sqrt{3+\sqrt{2}}}$.

75. $\dfrac{a+\sqrt{a}}{a\,(a+2)}\left[\dfrac{a-4}{\sqrt{a}-2}-\dfrac{(a+2)\sqrt{a}}{a+\sqrt{a}}\right]$.

[HINT: $a-4=(\sqrt{a}+2)(\sqrt{a}-2)$.]

76. Show that $\dfrac{2}{2-\sqrt{2}}=2+\sqrt{2}$.

77. Show that $\dfrac{2+\sqrt{2}-\sqrt{6}}{2-\sqrt{2}+\sqrt{6}}=\sqrt{5-2\sqrt{6}}$.

78. Show that $\left(\sqrt{24+16\sqrt{2}}-\sqrt{11+6\sqrt{2}}\right)^2=5.82842+$.

79. If $x=4+3\sqrt{5}$ and $y=5+2\sqrt{5}$, find value of $(x^2-y^2)^2$.

Solve:

80. $\sqrt{x+7}=\sqrt{12}$.

81. $\sqrt{x+3}=\sqrt{2x-10}$.

82. $3\sqrt{x-9}=2\sqrt{x+11}$.

83. $2\sqrt{x+3}=3\sqrt{10-x}$.

84. $\sqrt{x+13}=13-\sqrt{x}$.

85. $2(\sqrt{x}-3)(\sqrt{x}+3)=3$.

86. $\sqrt{x-3}-\sqrt{x-5}=\sqrt{2}$.

87. $\sqrt{x+3}+\sqrt{x-2}=5$.

88. $\sqrt{3}+\sqrt{9x+1}=3\sqrt{x}$.

89. $\sqrt{x+9mn}=3m+\sqrt{x}$.

90. $\sqrt[4]{10x-6}=\sqrt{2}$.

91. $\sqrt{x+3}+\sqrt{9x-1}=\sqrt{4x-1}$.

92. $\sqrt{2x+1}-\sqrt{8x-1}=\sqrt{2x-1}$.

93. $\sqrt{7+\sqrt{3+\sqrt{3x}}}=3$.

94. $\dfrac{\sqrt{x}+3}{\sqrt{x}-2}=\dfrac{\sqrt{x}+9}{\sqrt{x}+1}$.

95. $\sqrt{x+7}+\sqrt{x}=\dfrac{21}{\sqrt{x}}$.

96. $\sqrt{2+\sqrt{x^2-1}}=\sqrt{x-1}$.

97. $\dfrac{3}{\sqrt{x-3}}-\sqrt{x-3}=\sqrt{x}$.

98. $\sqrt{x}+\sqrt{x-2}-\dfrac{5}{\sqrt{x-2}}=0$.

99. $2\sqrt{x}=\dfrac{2}{\sqrt{4x-3}}+\sqrt{4x-3}$.

100. $\dfrac{\sqrt{x-5}+\sqrt{x}}{\sqrt{x-5}-\sqrt{x}}=-5.$

101. $\dfrac{\sqrt{x+m}-\sqrt{x}}{\sqrt{x+m}+\sqrt{x}}=\dfrac{1}{m}.$

102. $\dfrac{\sqrt{x+8}-\sqrt{x-8}}{\sqrt{x+8}+\sqrt{x-8}}=\dfrac{1}{4}.$

103. $\sqrt{2x-a^2}+\sqrt{2x+a^2}=2b.$

104. $\sqrt{x-2a}=\dfrac{3a}{\sqrt{x-2a}}-\sqrt{x}.$

105. $\sqrt{2x}-\sqrt{m}=\sqrt{2x-\sqrt{m(5m+8x)}}.$

106. $\sqrt{\dfrac{x-4}{x+4}}-\sqrt{\dfrac{x+4}{x-4}}=0.$

107. $\dfrac{3\sqrt{x}+2}{2\sqrt{x}-7}=\dfrac{3\sqrt{x}+1}{2\sqrt{x}-5}.$

108. $2\sqrt{x-2}-3\sqrt{x+2}=5\sqrt{x}.$

109. $\sqrt{x-\dfrac{a}{3}}+\dfrac{a}{\sqrt{x-\dfrac{a}{3}}}=\sqrt{x}.$

IMAGINARIES

Exercise 49

Simplify:

1. $\sqrt{-4}$.
2. $\sqrt{-16}$.
3. $\sqrt{-100}$.
4. $\sqrt{-144}$.
5. $2\sqrt{-81}$.
6. $\sqrt{-a^2}$.
7. $\sqrt{-a^4}$.
8. $-\sqrt{-9a^2}$.
9. $-2\sqrt{-36a^6}$.
10. $\sqrt{-(x+y)^2}$.
11. $-\sqrt{-(a+b)^2}$.
12. $-\dfrac{1}{m+n}\sqrt{-(m+n)^4}$.

Collect:

13. $\sqrt{-4}+\sqrt{-9}+\sqrt{-25}$.

14. $\sqrt{-9}+\sqrt{-36}-\sqrt{-49}+\sqrt{-64}$.

15. $\sqrt{-36}+\sqrt{-121}-\sqrt{-100}-\sqrt{-169}$.

16. $2\sqrt{-9}-3\sqrt{-16}+\sqrt{-49}-\sqrt{-25}$.

17. $3\sqrt{-25}-\sqrt{-81}-\sqrt{-144}+\sqrt{-256}$.

18. $\frac{1}{2}\sqrt{-4}+\frac{2}{3}\sqrt{-36}-\frac{1}{5}\sqrt{-25}+\frac{1}{6}\sqrt{-81}$.

19. $2\sqrt{-\frac{1}{4}}+3\sqrt{-\frac{4}{9}}-2\sqrt{-\frac{1}{25}}+3\sqrt{-\frac{1}{81}}$.

20. $\sqrt{-a^2}+2\sqrt{-4a^2}-\sqrt{-9a^2}+\sqrt{-16a^2}$.

21. $\sqrt{-4m^2}-\frac{1}{2}\sqrt{-16m^2}+\frac{2}{3}\sqrt{-121m^2}-\sqrt{-m^2}$.

22. $a\sqrt{-1}+\sqrt{-a^2}-\frac{2}{a}\sqrt{-a^4}+\sqrt{-16a^2}$.

Multiply:

23. $\sqrt{-2}$ by $\sqrt{-1}$.

24. $\sqrt{-3}$ by $\sqrt{-2}$.

25. $\sqrt{-3}$ by $\sqrt{3}$.

26. $2\sqrt{-3}$ by $\sqrt{-2}$.

27. $3\sqrt{-5}$ by $\sqrt{-3}$.

28. $\sqrt{-5}$ by $2\sqrt{-5}$.

29. $3\sqrt{-2}$ by $3\sqrt{2}$.

30. $-2\sqrt{-4}$ by $\sqrt{5}$.

31. $2\sqrt{-2}$ by $-\sqrt{6}$.

32. $3\sqrt{-8}$ by $2\sqrt{-4}$.

33. $3\sqrt{-27}$ by $-\sqrt{-3}$.

34. $-2\sqrt{5}$ by $3\sqrt{-8}$.

35. $a\sqrt{-a}$ by $\sqrt{a}$.

36. $-a\sqrt{-a}$ by $-\sqrt{-a}$.

37. $-2\sqrt{-3a}$ by $3\sqrt{-3}$.

38. $-3a\sqrt{-a}$ by $5\sqrt{-a^3}$.

39. $-2a\sqrt{-2a}$ by $a\sqrt{2a}$.

40. $-3a\sqrt{-a^3}$ by $-2a^2\sqrt{-a}$.

41. $\sqrt{-3}$ by $\sqrt{-6}$ by $2\sqrt{-2}$.

42. $\sqrt{-5}-\sqrt{-10}+2\sqrt{-15}$ by $2\sqrt{-5}$.

43. $(3+\sqrt{-2})$ by $(3-\sqrt{-2})$.

44. $(5-\sqrt{-2})(5+\sqrt{-2})$.

45. $(2-\sqrt{3})(2-\sqrt{-3})$.

46. $(2+\sqrt{-3})(2+2\sqrt{-3})$.

47. $(5-2\sqrt{-1})(3-3\sqrt{-1})$.

48. $(3\sqrt{2}-\sqrt{-2})(2\sqrt{2}-3\sqrt{-2})$.

49. $(4-3\sqrt{-2})(2+\sqrt{-2})$.

50. $(\sqrt{-3}-\sqrt{-2})(\sqrt{-3}+\sqrt{-2})$.

51. $(2\sqrt{-5}-3\sqrt{-2})(3\sqrt{-5}+2\sqrt{-2})$.

52. $(3\sqrt{-3}+2\sqrt{2})(4\sqrt{-3}-3\sqrt{2})$.

53. $(3-\sqrt{-2})^2$. 54. $(2-3\sqrt{-3})^2$. 55. $(2\sqrt{3}-2\sqrt{-3})^2$.

56. $(a-1-\sqrt{-1})(a-1+\sqrt{-1})$.

57. $(a+b\sqrt{-1})(a-b\sqrt{-1})$.

58. $(2a+2b\sqrt{-1})(2a-2b\sqrt{-1})$.

59. $(\sqrt{-1}+\sqrt{-2}+\sqrt{-3})(\sqrt{-1}+\sqrt{-2}-\sqrt{-3})$.

60. $(\sqrt{-2}-\sqrt{-3}+\sqrt{-1})(\sqrt{-2}+\sqrt{-3}-\sqrt{-1})$.

Rationalize the denominators of:

61. $\dfrac{3}{\sqrt{-2}}$. 64. $\dfrac{\sqrt{-15}}{2\sqrt{3}}$. 67. $\dfrac{3\sqrt{-9}}{-6\sqrt{-3}}$. 70. $\dfrac{-5\sqrt{12}}{-\sqrt{-3}}$.

62. $\dfrac{12}{\sqrt{-3}}$. 65. $\dfrac{\sqrt{-8}}{3\sqrt{-2}}$. 68. $\dfrac{\sqrt{10}}{\sqrt{-2}}$. 71. $\dfrac{\sqrt{-a^3}}{\sqrt{a}}$.

63. $\dfrac{\sqrt{-6}}{\sqrt{-2}}$. 66. $\dfrac{2\sqrt{-6}}{3\sqrt{-3}}$. 69. $\dfrac{\sqrt{15}}{2\sqrt{-3}}$. 72. $\dfrac{\sqrt{a^5}}{\sqrt{-a^3}}$.

73. $\dfrac{1}{1-\sqrt{-2}}$. 74. $\dfrac{3+\sqrt{-2}}{3-\sqrt{-2}}$. 75. $\dfrac{\sqrt{2}-\sqrt{-3}}{\sqrt{2}+\sqrt{-3}}$.

76. $\dfrac{\sqrt{3}+\sqrt{-2}}{1-\sqrt{-1}}$.

77. $\dfrac{\sqrt{-3}-\sqrt{-2}}{\sqrt{-3}+\sqrt{-2}}$.

78. $\dfrac{4\sqrt{-2}-3\sqrt{2}}{2\sqrt{-2}+3\sqrt{2}}$.

79. $\dfrac{a+\sqrt{-1}}{a-\sqrt{-1}}$.

80. $\dfrac{m+n\sqrt{-1}}{m-n\sqrt{-1}}$.

81. $\dfrac{\sqrt{2a}+2\sqrt{-2a}}{\sqrt{a}-\sqrt{-a}}$.

82. $\dfrac{2a+b\sqrt{-1}}{2a-b\sqrt{-1}}$.

83. $\dfrac{3a-2b\sqrt{-1}}{2a-b\sqrt{-1}}$.

84. $\dfrac{a-\sqrt{1-a}}{a-\sqrt{a-1}}$.

Simplify:

85. $(\sqrt{-1})^4$.

86. $(-\sqrt{-1})^5$.

87. $(\sqrt{-2})^4$.

88. $(-\sqrt{-2})^6$.

89. $(\sqrt{-1})^{-4}$.

90. $(-\sqrt{-1})^{-7}$.

91. $(1-\sqrt{-1})^2-(1-\sqrt{-1})^3$.

92. $(3-\sqrt{-2})^2-4(5-\sqrt{-3})^2$.

93. $(1+\sqrt{-1})^2-(1-\sqrt{-1})^2$.

94. $(1-\sqrt{-1})^4$.

95. $(1-\sqrt{-1})^3-(1-\sqrt{-1})^2+(1-\sqrt{-1})$.

96. $2(\sqrt{-1})^3-2\sqrt{-1}(\sqrt{-1}-1)^2$.

97. $\dfrac{2}{\sqrt{2}-1}-(1-\sqrt{-2})^2$.

98. $\dfrac{(1+\sqrt{-1})^2-(1-\sqrt{-1})^2}{2\sqrt{-1}}$.

99. $\dfrac{(x-\sqrt{-1})^2}{x+\sqrt{-1}}+\dfrac{(x+\sqrt{-1})^2}{x-\sqrt{-1}}$.

100. $\dfrac{m+n\sqrt{-1}}{m-n\sqrt{-1}}+\dfrac{m-n\sqrt{-1}}{m+n\sqrt{-1}}$.

Find the square root of:

101. $2-4\sqrt{-2}$.

102. $1-56\sqrt{-3}$.

103. $32-32\sqrt{-8}$.

104. $-3-12\sqrt{-3}$.

Resolve into imaginary factors:

105. $a+b$.

106. $a+4$.

107. $a+2b$.

108. a^2+4.

109. x^2+1.

110. $2x^2+3$.

GENERAL REVIEW

Exercise 50

Find H. C. F. and L. C. M. of:

1. $\begin{cases} m^3+6m^2+5m-12, \\ m^3-3m^2-22m-12. \end{cases}$

2. $\begin{cases} 6m^3-11m^2-14m+24, \\ 8m^3+18m^2-11m-30. \end{cases}$

3. $\begin{cases} 10a^{3x}+a^{2x}+18a^x-8, \\ 15a^{3x}-16a^{2x}-a^x+2. \end{cases}$

4. $\begin{cases} 2a^{6m}-3a^{4m}+3a^{2m}-1, \\ 8a^{6m}+4a^{4m}-2a^{2m}-1. \end{cases}$

5. $\begin{cases} 1-8x^{-3}, \\ 1+9x^{-1}-22x^{-2}, \\ 4x^{-3}-x^{-1}. \end{cases}$

6. $\begin{cases} x^{-7m}y^{5n}-x^{-5m}y^{7n}, \\ x^{-4m}+x^{-m}y^{3n}. \end{cases}$

7. $\begin{cases} 3a^{-1}-4a^{-\frac{1}{2}}-13+14a^{\frac{1}{2}}, \\ 3a^{-1}+5a^{-\frac{1}{2}}-10-42a^{\frac{1}{2}}. \end{cases}$

Simplify:

8. $\dfrac{2mn(m+n)^{-1}-m}{n^{-1}+(m-2n)^{-1}}+\dfrac{2mn(m+n)^{-1}-n}{m^{-1}+(n-2m)^{-1}}$.

9. $\dfrac{(a-1)[3a+(a-1)^2]^{-1}-(1-3a+a^2)(a^3-1)^{-1}-(a-1)^{-1}}{(1-2a+a^2-2a^3)(1+2a+2a^2+a^3)^{-1}}$.

10. $[(m-n)(m+n)^{-1}-(m+n)(m-n)^{-1}]$
$[mn^{-1}-nm^{-1}]^{-1}(m^{-2}-n^{-2})$.

11. $1+\dfrac{1}{2-\dfrac{3}{1+\dfrac{1}{2+\dfrac{3-2x}{x-1}}}}$.

12. If $\dfrac{m}{n}=\dfrac{x}{y}$, show that $\dfrac{m-n}{n}=\dfrac{x-y}{y}$.

13. $x+\dfrac{1}{x^2+x+\dfrac{x^4}{1-x^2}-\dfrac{x}{1-\dfrac{1}{x}}}$.

14. $(a-1)(a+3)^{-1}-1-(a-2)(a-3)^{-1}$
$+[5(3-2a)-a^2][9-a^2]^{-1}$.

15. $m^3+\dfrac{m^2}{m^2+\dfrac{1}{m^3-\dfrac{m^8+m^3-1}{m^5}}}$.

16. $[a-b(c-a^{-1})^{-1}]^{-1}$.

17. If $\dfrac{a}{b}=\dfrac{m}{n}$, show that $\dfrac{3a+b}{5a+3b}=\dfrac{3m+n}{5m+3n}$.

18. Show that

$$\frac{x^4+y^4+x^2y^2}{x^3-y^3}\div\left[\frac{x}{x-1}\left[x-1-\frac{y}{1+\dfrac{1}{x-1}}\right]\right]=1+\frac{xy}{(x-y)^2}.$$

Solve:

19. $\dfrac{3x+1}{2x^2+x-3}+\dfrac{x+5}{x-2x^2+6}=\dfrac{x+4}{2-3x+x^2}$.

20. $\dfrac{3x-1}{8}-\dfrac{x+1}{9x-16}+\dfrac{\frac{7}{6}x+4}{4}=\dfrac{4x+5}{6}$.

21. $\dfrac{x+1}{x+2}+(x-1)(x-2)^{-1}=\dfrac{2x-1}{x-1}$.

22. $\frac{1}{5}x+\frac{1}{3}(3-x)=\frac{1}{3}\{3x-\frac{1}{2}(6-x)\}-\dfrac{x-3}{15}$.

23. $5x-\dfrac{6}{(7x-12)^{-1}}=x-\left[3-\left\{x+\dfrac{2}{(3-x)^{-1}}\right\}\right]$.

24. $[1+x(1-x)^{-1}][1-x(1+x)^{-1}][1-x^2+(1-x^2)x^{-1}]=3$.

25. Show that $-\dfrac{4a}{5}$ satisfies $\dfrac{a-4x}{2a-x}-\dfrac{a+2x}{x+2a}=2$.

26. Solve $(m+x)(n+x)-m(n+p)=\frac{m^2p}{n}+x^2$.

27. Show that $cd+\frac{x}{d}=\frac{cx}{a}+a$ when $x=ad$.

Solve the following:

28. $\begin{cases} \frac{x-5}{4}-\frac{2x-y-1}{3}=\frac{2y-2}{5}, \\ \frac{2y+x-1}{9}=\frac{x+y}{4}. \end{cases}$

29. $\begin{cases} \frac{x-2}{3}-\frac{x+y-1}{4}=3, \\ \frac{x+3}{4}-\frac{x-2y-1}{2}=1. \end{cases}$

30. $\begin{cases} \frac{2x+1}{3}-\frac{2-3y}{5}=\frac{1}{6}, \\ \frac{3y-2}{4}-\frac{2x+3}{6}=\frac{3}{8}. \end{cases}$

31. $\begin{cases} \frac{4x-3y+1}{2}=\frac{x+3y}{5}, \\ \frac{3x+2y}{5}-1=\frac{2x-3y}{3}. \end{cases}$

32. $\begin{cases} \frac{4x-1+y}{3}-2y=\frac{x+1}{2}, \\ \frac{x-2}{4}-2(x-y)=\frac{3}{2}. \end{cases}$

33. $2x+3y=3x-2y=1$.

34. $\begin{cases} ax-by=a^2-b^2-2ab, \\ bx+ay=2ab+a^2-b^2. \end{cases}$

35. $\begin{cases} \frac{x}{a+b}+\frac{y}{a-b}=2a, \\ \frac{x-y}{4ab}=1. \end{cases}$

36. $\begin{cases} \frac{1}{2}(x+y)=1+\frac{x-y}{2a}, \\ \frac{a}{2}(x-y)=1+\frac{x-y}{2a}. \end{cases}$

37. $\begin{cases} \frac{3}{2x}+\frac{4}{3y}=-\frac{13}{3}, \\ \frac{5}{3x}-\frac{3}{2y}=\frac{31}{27}. \end{cases}$

38. $\begin{cases} \frac{1}{y}-\frac{1}{x}=\frac{2}{3a}, \\ \frac{1}{x}+\frac{1}{y}=\frac{3}{2a}. \end{cases}$

39. $\begin{cases} 3x+4y-5z=2, \\ 2x-3y-8z=-9, \\ 3x-y-2z=0. \end{cases}$

40. $\begin{cases} 2x-y+3z=-\frac{5}{6}, \\ 3x-3y-2z=\frac{3}{2}, \\ 4x+2y-5z=5\frac{1}{6}. \end{cases}$

41. $\begin{cases} cy+bz=2, \\ az+cx=2, \\ bx+ay=2. \end{cases}$

42. $\begin{cases} x^{-1}-y^{-1}-z^{-1}=3, \\ x^{-1}-2y^{-1}-3z^{-1}=11, \\ y^{-1}-x^{-1}-2z^{-1}=3. \end{cases}$

Expand:

43. $(2x-a)^7$.

44. $[3a-\{2a+(5a-\overline{2a-4})\}]^4$.

45. $\left(\frac{x^6-64}{x^4+4x^2+16}\right)^5$.

46. $(a^{-1}+2x^{-1})^4$.

47. $\left(x^{-2}-\frac{3x}{a^{-1}}\right)^6$.

48. $\left(\sqrt{x^{-3}}-\frac{3}{\sqrt{ax}}\right)^5$.

Find the square root of:

49. $x^{-4m}+4x^{-3m}-2x^{-2m}-12x^{-m}+9$.

50. $a^{4m}-2a^{2m}-11+12a^{-2m}+36a^{-4m}$.

51. $4-4a^{\frac{1}{2}}-11a+14a^{\frac{3}{2}}+5a^2-12a^{\frac{5}{2}}+4a^3$.

52. $(x^{-2}-4)(x^{-2}-3x^{-1}+2)(x^{-2}+x^{-1}-2)$.

53. $(m+m^{-1})^2-4(m-m^{-1})$.

Find the numerical value of:

54. $\sqrt{.073}$ to 5 decimal places.

55. $\sqrt{.0073}$ to 5 decimal places.

56. $\sqrt{5-3\sqrt{.025}}$ to 3 decimal places.

57. $\sqrt{.007+.3\sqrt{14.4}}$ to 3 decimal places.

Simplify:

58. $\frac{a^{\frac{1}{2}}\sqrt[3]{x^{-2}}}{x^{\frac{1}{3}}\sqrt[3]{a^{-2}}}\cdot\sqrt[3]{-27ax^4}$.

59. $[-3\sqrt[3]{-27a^{-2}}\div(-8^{-1})^{-\frac{1}{3}}]^{-1}$.

60. $3x^0-(3x)^0-\left(\frac{1}{3}\right)^{-1}+3^0$.

61. $(81x^{16})^{-\frac{1}{4}}\cdot\left(\frac{1}{-3x^0}\right)^{-2}$.

62. $\left[\sqrt[3]{\frac{-8a^{-4}b^2}{c^{-1}\sqrt{d}}}\div\sqrt[4]{\frac{16a}{c^3}}\right]^{-1}$.

63. $(x^{a-3})^c\cdot(x^{2-3a})^{2c}\cdot(x^c)^{1-4a}$.

64. $\left(\sqrt[3]{-8c^5d^{-1}}\cdot\sqrt{\frac{\sqrt{c^{-1}d}}{16ac^2\sqrt{d}}}\right)$.

65. $64^{-\frac{5}{6}}\cdot\left(\frac{1}{8}\right)^{-\frac{5}{3}}\cdot\sqrt[4]{81c^{-8}}$.

66. $(m^3)^{\frac{n+1}{6}}(m^2)^{\frac{n-1}{4}}\div(m^{-1})^{-n}$.

67. $(8a\sqrt[3]{x^2}\div 27x^{\frac{1}{3}}\sqrt{a^{-2}x})^{-\frac{2}{3}}$.

68. $(x^m)^{n-1}\cdot(x^n)^{m+1}(x^m)^{1-2n}$.

69. $\sqrt[4]{\frac{n^{-\frac{1}{3}}a^3}{a^{-1}\sqrt{x}}}\cdot\sqrt[3]{\frac{x^{\frac{1}{2}}\sqrt{a^3n}}{x^{\frac{3}{4}}a^{\frac{3}{4}}\sqrt[3]{n^5}}}$.

70. $\sqrt{25a^{-1}b\sqrt{x^{-3}}}(27^{-1}a^{-\frac{1}{2}}x^{-\frac{1}{3}}\sqrt{ax^{\frac{4}{3}}b^{-1}})^{\frac{1}{3}}$.

71. $[7x^{-2}y^{-\frac{1}{2}}]^0\cdot[3x^0y^{-\frac{2}{3}}z]^{-\frac{3}{2}}\div\left(\frac{1}{3z^{-1}}\right)^2$.

Collect:

72. $\left(\frac{3}{2}\right)^{-2}+\frac{5}{3^{-2}}-\left(\frac{1}{16}\right)^{-\frac{1}{4}}-(-3)^2$.

73. $7^{-1}+\left(\frac{2}{5}\right)^{-3}-(7x)^0-49^{-\frac{1}{2}}-2^{-2}$.

74. $81^{-\frac{3}{4}}-5x^0+9(3)^{-3}+(125^{-1})^{-\frac{1}{3}}+1^{-\frac{3}{5}}$.

75. $\left(\frac{\sqrt[3]{3}}{3^{-\frac{4}{5}}}\right)^3-\sqrt{3\cdot 27^{-1}}+(-243)^{\frac{2}{5}}+\left(\frac{\sqrt{3^{-1}}}{2^{-1}}\right)^2$.

Multiply:

76. $\frac{1}{a^2}-1+b^2$ by $a^{-2}+1+\frac{1}{b^{-2}}$.

77. $a^3-3+3a^{-3}-a^{-6}$ by $a^2-\frac{2}{a}+\frac{1}{a^4}$.

78. $\frac{1}{\sqrt{x^3}}-4x^{-1}+\frac{11}{\sqrt{x}}-24$ by $\frac{1}{x}+\frac{4}{\sqrt{x}}+5$.

Divide:

79. $a^{-1}-b$ by $a^{-\frac{1}{2}}-b^{\frac{1}{2}}$.

80. $27m^{-3}-8n^2$ by $3m^{-1}-2n^{\frac{2}{3}}$.

81. $x^{\frac{2}{3}}+2x^{\frac{1}{3}}-16x^{-\frac{2}{3}}-32x^{-1}$ by $x^{\frac{1}{3}}+4x^{-\frac{1}{3}}+4x^{-\frac{2}{3}}$.

82. $a^{-4m}+a^{-2m}b^{2n}+b^{4n}$ by $\frac{1}{a^{2m}}-\frac{b^n}{a^m}+b^{2n}$.

Simplify: 83. $(a^m+2\,a^{-m})^2-(a^m-2\,a^{-m})^2$.

84. $[(a+b)^{\frac{1}{2}}+(a-b)^{\frac{1}{2}}]^2$.

85. $[(m-1)^{\frac{1}{3}}+(m+1)^{\frac{1}{3}}]^3$.

86. $(m^{\frac{3x}{2}}-m^{\frac{-3x}{2}})\div(m^x+1+m^{-x})$.

Collect: 87. $\frac{3}{7}\sqrt{16\frac{1}{3}}+\frac{15}{\sqrt{3}}-\frac{1}{12}\sqrt{432}+\frac{3}{20}\sqrt{1000}$.

88. $5\sqrt{75}-\frac{1}{3}\sqrt{147}-\frac{6}{\sqrt{3}}+2\sqrt{\frac{1}{3}}-\sqrt[3]{81}$.

89. Multiply $\sqrt[4]{\frac{12}{35}}$ by $\sqrt[3]{\frac{245}{36}}$ by $\sqrt{\frac{3}{7}}$.

Simplify: 90. $(3\sqrt{6}-2\sqrt{3}+5\sqrt{2})^2$.

91. $(2\sqrt{5}-3\sqrt{3}-2\sqrt{2})(2\sqrt{3}+\sqrt{5}-3\sqrt{2})$.

92. $(\sqrt[4]{9}-6\sqrt{\frac{1}{8}}+\sqrt{48})^2$.

93. $\left(\sqrt[6]{27}-\sqrt{6\tfrac{3}{4}}+\frac{2}{\sqrt{2}}\right)^2$.

94. $(\sqrt{3}-\sqrt{2})^2-2\,(2-\sqrt{6})\,(2-\sqrt{2})$.

95. $(3\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})(3\sqrt{3}-\sqrt{2})$.

96. $\left(\frac{2\sqrt{5}}{\sqrt{10}}-\frac{8\sqrt{3}}{\sqrt{6}}+\frac{10}{\sqrt{2}}\right)(\sqrt[4]{25}-\sqrt[6]{216})$.

97. $\sqrt{52-6\sqrt{35}}$.

98. $\sqrt{2\,a+1-2\sqrt{a^2+a}}$.

99. $(\sqrt{2}-\sqrt{5})^2$.

100. $(\sqrt{2}-\sqrt[3]{3})^2$.

101. $(2\sqrt{2}-3\sqrt[3]{2})^2$.

102. $(\sqrt{2}+\sqrt[3]{3}-\sqrt[4]{6})^2$.

103. $\frac{3\sqrt[3]{9}\cdot 4\sqrt[3]{81}}{\sqrt[3]{243}}$.

Rationalize the denominators of the following:

104. $\dfrac{\sqrt{2}-4}{3\sqrt{6}-2\sqrt{3}}$.

105. $\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$.

106. $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}+1}$.

107. $\dfrac{b\sqrt{a}+a\sqrt{b}}{b\sqrt{a}-a\sqrt{b}}$.

Find the numerical value of:

108. $\sqrt{\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}}$.

109. $\dfrac{\sqrt{2}}{\sqrt{3}-2\sqrt{2}}$.

110. Which is the greater, $\sqrt{5}$ or $\sqrt[3]{11}$?

111. Which is the greatest, $\sqrt{\frac{4}{3}}$, $\sqrt[3]{\frac{3}{2}}$, or $\sqrt[4]{\frac{7}{4}}$?

112. Show that $\sqrt{\frac{3}{2}} > \sqrt[3]{\frac{5}{3}}$.

Simplify:

113. $\left(\dfrac{n}{3}-\dfrac{\sqrt{m^2-n^2}}{3}\right)^2$.

114. $\left(\sqrt{x^2-1}+\dfrac{1}{\sqrt{x^2-1}}\right)^4$.

115. If $m=\frac{1}{2}(\sqrt{c+d}+\sqrt{c-2d})$ and $n=\frac{1}{2}(\sqrt{c+d}-\sqrt{c-2d})$, find the value of m^2+n^2.

116. Simplify by inspection:

$$(\sqrt{m+n}+\sqrt{m}-\sqrt{n})\,(\sqrt{m+n}-\sqrt{m}-\sqrt{n}).$$

117. Change $\dfrac{c+d}{c-d}\sqrt{\dfrac{c-d}{c+d}}$ to an entire surd.

118. If $a=11$, $b=-12$, and $c=3$, what is the numerical value of

$$\frac{-b-\sqrt{b^2-4ac}}{2a}?$$

119. If $n=11$, $a=5\sqrt{2}-2\sqrt{3}$, $d=-(\sqrt{2}+\sqrt{3})$, find value of $\dfrac{n}{2}[2a+(n-1)d]$.

Simplify:

120. $\dfrac{\left(1+\sqrt{\dfrac{x-1}{x+1}}\right)^2}{1-\dfrac{x-1}{x+1}}-\sqrt{x^2-1}.$

121. $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}+\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}.$

122. $\dfrac{\sqrt{x^2+9}+\sqrt{x^2-9}}{\sqrt{x^2+9}-\sqrt{x^2-9}}+\dfrac{\sqrt{x^2+9}-\sqrt{x^2-9}}{\sqrt{x^2+9}+\sqrt{x^2-9}}.$

123. $\left(\dfrac{\sqrt{a-b}-\sqrt{a+b}}{\sqrt{a-b}+\sqrt{a+b}}-\dfrac{\sqrt{a-b}+\sqrt{a+b}}{\sqrt{a-b}-\sqrt{a+b}}\right)^2.$

Solve the following equations:

124. $\sqrt{x}+\sqrt{x-5}=\sqrt{5}.$

125. $\sqrt{3x-2}+\sqrt{3x}-2=0.$

126. $\sqrt{x+6}+\sqrt{x-4}=2.$

127. $\sqrt{x+2}+\sqrt{4x+1}=\sqrt{9x+7}.$

128. $\sqrt{8x+1}-\sqrt{2x+3}-\sqrt{2x-2}=0.$

129. $\sqrt{2+\sqrt{4x^2+5}}=\sqrt{2x+3}.$

130. $\dfrac{1}{\sqrt{x-2}}+\dfrac{1}{\sqrt{x+2}}=\dfrac{2}{\sqrt{x^2-4}}.$

131. $\dfrac{4}{\sqrt{4+x}}-\sqrt{x}-\sqrt{4+x}=0.$

132. $\dfrac{1}{x+\sqrt{x^2-3}}+\dfrac{1}{x-\sqrt{x^2-3}}=\dfrac{1}{3}.$

133. $\dfrac{2-x}{\sqrt{2}+\sqrt{x}}=\dfrac{x}{\sqrt{2}-\sqrt{x}}$.

134. $\dfrac{2m-n}{\sqrt{2mx}-n}=\dfrac{\sqrt{2mx}+n}{x+n}$.

Simplify:

135. $(\sqrt{-1})^3+(\sqrt{-1})^4$.

136. $(\sqrt{-1})^5-(\sqrt{-1})^6$.

137. $(1-\sqrt{-1})^2+(1+\sqrt{-1})^2$.

138. $(1-2\sqrt{-1})^2+2(2-\sqrt{-1})^2$.

139. $(\sqrt{3}+\sqrt{-2})(\sqrt{3}-2\sqrt{-2})$.

140. $(2\sqrt{3}-3\sqrt{-2})^2$.

141. $(-\frac{1}{2}+\frac{1}{2}\sqrt{-3})^3$.

142. $\sqrt{4\sqrt{6}-11}$.

Rationalize the denominators of:

143. $\dfrac{\sqrt{-8}}{\sqrt{-2}}$.

145. $\dfrac{2\sqrt{18}}{-\sqrt{-2}}$.

147. $\dfrac{2}{\sqrt{5}-\sqrt{-3}}$.

144. $\dfrac{\sqrt{-18}}{\sqrt{2}}$.

146. $\dfrac{-\sqrt{28}}{3\sqrt{-7}}$.

148. $\dfrac{\sqrt{-2}-3\sqrt{-3}}{\sqrt{-2}+2\sqrt{-3}}$.

What are the conjugate imaginary factors of:

149. $m+2n$?

150. a^2+7?

151. $3m^2+2$?

QUADRATIC EQUATIONS

I. NUMERICAL QUADRATICS

Exercise 51

Solve:

1. $2x^2 - 7x = 15$.
2. $2x^2 + x = 15$.
3. $3x^2 + 7x = 20$.
4. $6x^2 - 19x = 36$.
5. $5x^2 + 14x = 3$.
6. $x^2 + 3x + 1 = 0$.
7. $x^2 + 3x + 3 = 0$.
8. $x^2 - 5x - 1 = 0$.
9. $x^2 - 5x + 7 = 0$.
10. $3x^2 + 2x + 1 = 0$.
11. $x^2 - 9 = 0$.
12. $x^2 - 4x = 0$.
13. $x^2 + 1 = 0$.
14. $3x^2 = 7$.
15. $5x^2 = 11x$.
16. $2x^2 + 3 = 0$.
17. $5x^2 - 3x + 1 = 0$.
18. $3x^2 + 5x + 3 = 0$.
19. $5x^2 = 2x + 1$.
20. $7x^2 = 6x - 1$.
21. $3x^2 + x - 5 = x^2 - 11 + 8x$.
22. $(2x + 3)(x - 5) = (x - 5)(x + 3)$.
23. $(3x - 7)(2x + 1) = (5x + 2)(2x - 3)$.
24. $(2x - 1)(3x + 5) - (x + 5)(3x - 2) = 5 - (x - 2)^2$.
25. $(3x + 1)(x - 5) - (2x - 1)(3x + 2) = (x + 6)^2 - 1$.
26. $(x - 5)^2 - (2x - 3)^2 - (x + 4)^2 = x(x - 5)$.
27. $(2x + 1)(x - 5) + 2(x - 3)^2 - x(x - 4) = 2(x - \frac{3}{2})^2 - 15\frac{1}{2}$.
28. $(x - 5)^2 - (3 - 2x)^2 - (2x - 1)(x + 4) + 5x = 0$.
29. $2(x + 2)(3x - 1) - 3(x + 1)(4 - x) = x(x - 2) - 17$.

30. $\frac{1}{x-1}+\frac{2}{x+1}=1.$

31. $\frac{x-1}{x+1}-\frac{1}{x}=-\frac{1}{6}.$

32. $\frac{1}{3-x}-\frac{4}{5}=\frac{1}{9-2x}.$

33. $\frac{x+2}{x-2}-\frac{x-2}{x+2}=1\frac{1}{2}.$

34. $\frac{2x-1}{2x+1}+\frac{2x+3}{x+8}=2.$

35. $\frac{4x-1}{2x+1}+\frac{3}{2}-\frac{1}{x}=0.$

36. $\frac{x}{x-3}-\frac{x-1}{x+3}=2.$

37. $\frac{x+3}{x+5}+\frac{3x-2}{x-5}=1.$

38. $\frac{2x-1}{x}-\frac{x-2}{x+1}=3.$

39. $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3.$

40. $\frac{x+1}{x+2}-1=\frac{x+3}{x+4}.$

41. $\frac{2x-3}{3x-2}-\frac{x-1}{x}=-1.$

42. $\frac{1}{3}-\frac{1}{x}=1-\frac{x+2}{2x-1}.$

43. $\frac{5}{x-1}-\frac{2}{x+1}+\frac{5}{2-x}=0.$

44. $\frac{3}{2x-7}-\frac{8}{x+4}+\frac{5}{x+7}=0.$

45. $\frac{2}{x-2}=\frac{10}{x+2}+1-\frac{5}{x^2-4}.$

46. $\frac{x-3}{1-x}-\frac{2+x}{1+x}=\frac{(x+1)^2+4}{1-x^2}.$

47. $\frac{2x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-3}{x-4}.$

48. $\frac{2x-1}{x+1}-\frac{x-7}{x-1}=4-\frac{3x-1}{x+2}.$

49. $\frac{x+1}{x-3}-\frac{5x+2}{x+3}+\frac{x^2-3}{9-x^2}=0.$

50. $\frac{2x+1}{2x-3}-1-\frac{x-4}{2x+3}=\frac{-7x}{9-4x^2}.$

51. $2-\frac{x-3}{x-5}-\frac{x+13}{x-3}=\frac{13}{2x+5}.$

52. $\frac{5}{2x-1}-\frac{x-1}{x+1}=\frac{2(x+3)}{2x+3}-2.$

53. $\frac{x+1}{3x+2}=\frac{2x-3}{3x-2}-1-\frac{36}{4-9x^2}.$

54. $3\sqrt{x+2}=2x-5.$

55. $x+5=2\sqrt{5x+1}.$

56. $\sqrt{3x+7}-x=3.$

57. $2\sqrt{3x+4}=\sqrt{5x^2-3x-4}.$

58. $2\sqrt{x}=x-3.$

59. $2\sqrt{x-1}=x-4.$

60. $2\sqrt{x}=\sqrt{x-3}+3.$

61. $\sqrt{2x}-1=\sqrt{x+1}.$

62. $\sqrt{x+1}-1=\sqrt{2x-5}.$

63. $2\sqrt{x+1}-\sqrt{2x+3}=1.$

64. $2\sqrt{3x+7}+1=3\sqrt{x+3}.$

65. $5\sqrt{x-1}-3\sqrt{3x-2}=-1.$

66. $2\sqrt{3x-2}-3\sqrt{x+3}+1=0.$

67. $\sqrt{x+1}+\sqrt{3x+4}=\sqrt{5x+6}.$

68. $\sqrt{x+5}-\sqrt{2x-7}=\sqrt{x}.$

69. $2\sqrt{x+1}+\sqrt{x-2}=\sqrt{7x+4}.$

70. $\sqrt{4x-3}-\sqrt{2x+2}=\sqrt{x-6}.$

71. $\sqrt{3x-5}+\sqrt{x-9}=2\sqrt{x-1}.$

72. $\sqrt{2-3x}-\sqrt{7+x}=\sqrt{5+4x}.$

73. $\sqrt{3x+2}-\sqrt{2x+1}=\sqrt{x+1}.$

74. $\sqrt{2x+3}-\sqrt{8x+5}=-\sqrt{4x-1}.$

75. $2\sqrt{3x+2}-\sqrt{6x-3}=3\sqrt{3x-1}.$

76. $\sqrt{3x}+\sqrt{x-2}=\dfrac{4}{\sqrt{x-2}}.$

77. $\sqrt{3x+1}-\sqrt{2x}=\dfrac{5}{\sqrt{3x+1}}.$

78. $\sqrt{3x+3}-\sqrt{x-1}=\dfrac{2}{\sqrt{x-1}}.$

79. $3\sqrt{x+4}-\sqrt{2x-9}=\dfrac{8}{\sqrt{2x-9}}.$

80. $\sqrt{3x-5}+\dfrac{3}{\sqrt{x-1}}=2\sqrt{x-1}.$

81. $\sqrt{x+2}=\dfrac{14}{\sqrt{x-3}}-2\sqrt{x-3}.$

82. $\dfrac{3}{2+\sqrt{x-1}}+\dfrac{1}{2-\sqrt{x-1}}=2.$

83. $\dfrac{1}{x-\sqrt{x^2-5}}-\dfrac{1}{x+\sqrt{x^2-5}}=\dfrac{x+1}{5}.$

84. $\dfrac{7}{x+\sqrt{x^2+7}}-\dfrac{7}{x-\sqrt{x^2+7}}=3x-1.$

II. LITERAL QUADRATICS

Exercise 52

Solve:

1. $2x^2-5ax=3a^2.$
2. $6a^2x^2-7ax=20.$
3. $3x^2-abx-2a^2b^2=0.$
4. $5a^2x^2-24b^2=26abx.$
5. $18b^2x^2=3bcx+10c^2.$
6. $x^2-2ax+a^2=4.$
7. $x^2-a^2+6x+9=0.$
8. $4x^2=4ax-a^2+1.$
9. $ax^2=3x+4a-6.$
10. $ax^2+ax+2=2a-x.$
11. $2x^2+a^2=x+3ax+1.$
12. $x^2+ax-2x+1=2a^2+a.$
13. $2a^2x^2-a^2x-9ax=a^2-9.$
14. $6x^2+ax+8x=a^2+a-2.$
15. $a^2x^2-x^2+2bx=b^2.$
16. $b^2x^2+abx-4x^2=2a(3x+a).$
17. $a^2x^2-x-ax^2-ax=(a+1)^2.$
18. $4x^2-4ax=b-a^2+c.$
19. $4a^2x(x+1)+(a-1)(a+1)=0.$
20. $a^2x^2-b^2=ax^2+bx.$
21. $a^2x^2-(b-1)^2=0.$
22. $4x^2-a^2=2a+1.$
23. $9a^2x^2-c^2+6c=9.$

24. $2ax^2 - bx = cx.$

25. $3ax^2 + 4bx + 5c = 0.$

26. $ax^2 + 2bx + 3c = 0.$

27. $x^2 + px + q = 0.$

28. $lx^2 - mx - lm = 0.$

29. $x^2 - 2ax - 2x + a^2 + 1 = 0.$

30. $(a^2 - b^2)x^2 = a^2(2x - 1).$

31. $(1 - a^2)(x + a) = 2a(1 - x^2).$

32. $\dfrac{a}{x-1} - bx = -a.$

33. $\dfrac{2x^2}{a-1} - x - a - \dfrac{x}{a-1} + 2 = 0.$

34. $\dfrac{x}{a+2} - \dfrac{a-1}{x} = \dfrac{x+2}{x(a+2)}.$

35. $\dfrac{1}{a+x} + \dfrac{1}{b+x} = \dfrac{a+b}{ab}.$

36. $\dfrac{1}{x-a+b} = \dfrac{1}{x} - \dfrac{1}{a} + \dfrac{1}{b}.$

37. $\dfrac{2a+x}{2a-x} + \dfrac{a-2x}{a+2x} = 2\tfrac{2}{3}.$

38. $\dfrac{a-b}{x} - \dfrac{2bx}{x-1} - 2a = 0.$

39. $\dfrac{1}{a-x} - \dfrac{1}{b-x} = \dfrac{1}{a} - \dfrac{1}{b}.$

40. $\dfrac{x-a}{x+b} = \dfrac{2a}{b} + \dfrac{2x+3a}{2x-b}.$

41. $\dfrac{18a}{5a-x-3b} - 1 = \dfrac{x+a}{x+2b}.$

42. $ax^2 - \dfrac{ax(a+b)}{a-b} + bx^2 = \dfrac{ab}{a-b} - bx.$

43. $x - \dfrac{1}{x} = \dfrac{3a^2 - 2a}{(a-1)(2a-1)}.$

44. $\dfrac{1}{x+a} + \dfrac{1}{x+b} - \dfrac{1}{x+c} = 0.$

45. $\sqrt{2x^2 - ax - a + 2} = a - 2.$

46. $\sqrt{3x^2 - 4ax + 1} = 2(x - a).$

47. $\sqrt{x-a} + \sqrt{3x - 2a} = \sqrt{2x + 5a}.$

48. $\sqrt{x} + \sqrt{a - \sqrt{ax + x^2}} = \sqrt{a}.$

49. $\sqrt{x-a} - \dfrac{b}{\sqrt{x-a}} = \dfrac{a}{\sqrt{x-b}} - \sqrt{x-b}.$

50. $\sqrt{b+x} - \sqrt{b-x} = \sqrt{b}.$

51. $4x(\sqrt{a} - x) = a - b.$

52. $\sqrt{x+a^2}+\sqrt{x-2\,a^2}=\sqrt{3\,x}$.

53. $\sqrt{3\,x^2-4\,ax+1}=2\,(a-1)$.

54. $\sqrt{2\,a^2x^2-6\,ax-a^2+5}=a-1$.

55. $\sqrt{x-a}-\sqrt{ax}=\sqrt{x+a}$.

56. $\sqrt{ax-b}-\sqrt{ax+b}=\sqrt{a^2x+2\,ax-ab}$.

III. EQUATIONS IN THE QUADRATIC FORM

Exercise 53

Find all of the values of x:

1. $x^4-13\,x^2+36=0$.
2. $4\,x^4-29\,x^2+25=0$.
3. $9\,x^4-28\,x^2+3=0$.
4. $9\,x^4+29\,x^2=80$.
5. $x^4-16=0$.
6. $x^3+8=0$.
7. $x^3-x=0$.
8. $2\,x^4-x^2=15$.
9. $x^4-x=0$.
10. $x^4+64\,x=0$.
11. $x^4=7\,x^2+8$.
12. $8\,x^3=27$.
13. $x+4\sqrt{x}=5$.
14. $2\,x^{\frac{1}{2}}-5\,x^{\frac{1}{4}}=3$.
15. $3\,x^{\frac{2}{3}}+4\,x^{\frac{1}{3}}=4$.
16. $9\,x^{\frac{4}{3}}-37\,x^{\frac{2}{3}}+4=0$.
17. $3\,x+5\,x^{\frac{1}{2}}=12$.
18. $12\,x^{\frac{2}{3}}-11\sqrt[3]{x}=15$.
19. $x^6-7\,x^3=8$.
20. $4\,x^{\frac{4}{5}}-17\sqrt[5]{x^2}+4=0$.
21. $x^{\frac{6}{5}}+26\sqrt[5]{x^3}=27$.
22. $x^4+x^2+1=0$.
23. $x^{-1}+x^{-\frac{1}{2}}-6=0$.
24. $2\,x^{-4}-5\,x^{-2}=12$.
25. $3\,x^{-\frac{1}{2}}+7\,x^{-\frac{1}{4}}=6$.
26. $2\sqrt[3]{x^{-2}}-7\,x^{-\frac{1}{3}}=4$.
27. $9\,x^{-\frac{4}{3}}+4=37\sqrt[3]{x^{-2}}$.
28. $2\,x^{-\frac{4}{3}}-5\sqrt[3]{x^{-2}}=3$.

29. $8\sqrt[5]{x^{-6}} = 15\sqrt[5]{x^{-3}} + 2.$

30. $3x^{-1} = 14 - 19\sqrt{x^{-1}}.$

31. $4x^{-\frac{1}{2}} + 4x^{-\frac{1}{4}} = 3.$

32. $(x+1)^2 = 3(x+1) + 40.$

33. $(x^2+3x)^2 - 2(x^2+3x) = 8.$

34. $(x^2-4x)^2 - 9(x^2-4x) = 36.$

35. $(x-1)^4 - 13(x-1)^2 + 36 = 0.$

36. $\sqrt{3x-5} - 3\sqrt[4]{3x-5} = 10.$

37. $2\sqrt{5x+1} + \sqrt[4]{5x+1} = 6.$

38. $\sqrt[3]{2x-1} + 3\sqrt[6]{2x-1} = 4.$

39. $2(x+1)^{\frac{2}{3}} - 3(x+1)^{\frac{1}{3}} = 2.$

40. $(4x+3)^{\frac{2}{5}} - \sqrt[5]{4x+3} = 6.$

41. $3\sqrt{3x+1} + \sqrt[4]{3x+1} = 14.$

42. $2(2x+3) - 5\sqrt{2x+3} = 3.$

43. $2x^2 + 1 - 2\sqrt{2x^2+1} = 3.$

44. $x^2 - x + 4 - 6\sqrt{x^2-x+4} + 8 = 0.$

45. $(x^2-x)^2 - x^2 + x = 30.$

46. $3x^2 - 4x + 3\sqrt{3x^2-4x+2} = 2.$

47. $2x^2 + 3x + \sqrt{2x^2+3x+7} = 5.$

48. $x^2 = 8 - 3x - 4\sqrt{x^2+3x-3}.$

49. $3x^2 - x = 6\sqrt{3x^2-x-6} + 22.$

50. $x^2 = 5x + 10 - 2\sqrt{x^2-5x-2}.$

51. $2x^2 + x + 5 = 5\sqrt{2x^2+x+1}.$

52. $(x^2-x+1)^2 = 3x^2 - 3x + 1.$

53. $x^2 - x + 5\sqrt{2x^2-5x+6} = \frac{3}{2}(x+11).$

54. $\sqrt{2x^2+9x+9} + \sqrt{2x^2+7x+5} = \sqrt{2}.$

55. $x(2x-3)(2x^2+13x+20) = 0.$

56. $ax(x-1)(x^2+1)(x^3-8) = 0.$

57. $(x^2-x-12)(x^2+x-90)(x^2+x-110) = 0.$

SIMULTANEOUS QUADRATICS

Exercise 54

Solve:

1. $2y-3x=7$; $3x^2-4xy-4y^2=15$.
2. $x^2+y^2=58$; $xy=21$.
3. $3x^2-2xy=24$; $5x^2-4y^2=44$.
4. $x-2y=1$; $xy=3$.
5. $x^2+xy+y^2=13$; $x^2-xy+y^2=7$.
6. $2x-y=7$; $5x^2-3y^2=-7$.
7. $3x^2-5y^2=28$; $3xy-4y^2=8$.
8. $2x^2+xy-y^2=5$; $x^2+xy-y^2=1$.
9. $3x+4y=2$; $x^2-xy-5y^2=1$.
10. $x+3y=-4$; $6x^2+13xy-5y^2=21y-12x+18$.
11. $x+y=7$; $x^2+y^2=29$.
12. $x^3+y^3=218$; $x+y=2$.
13. $xy+3y^2=20$; $x^2-3xy=-8$.
14. $x+3\sqrt{y}+4=0$; $2x^2-5y^2=5$.
15. $2x-3y=3$; $4x^2-15-7xy=0$.
16. $x^2-xy+y^2=21$; $x^3+y^3=189$.
17. $3x^2-5y^2=7$; $4xy-y^2=7$.
18. $6x-8y+23=0$; $3y^2-5xy-2x=26$.
19. $x-y=1$; $x^3-y^3=\frac{1}{8}$.

20. $2x^2+3xy-4y^2=10$; $7x-5y=9$.

21. $x^2-3y^2=1\frac{1}{2}$; $2x^2+y^2=4\frac{3}{4}$.

22. $\frac{1}{xy}=\frac{3}{4}$; $\frac{1}{x}-\frac{1}{y}=1$.

23. $x^2-xy+y^2=3\frac{1}{4}$; $x^3+y^3=3\frac{1}{4}$.

24. $(x+y)^2-5(x+y)=36$; $9x-4y=29$.

25. $\frac{1}{x}-\frac{1}{y}=5$; $\frac{1}{x^2}+\frac{1}{y^2}=13$.

26. $3x+2y=13$; $x^2y^2-xy=30$.

27. $3x^2+5y^2=17$; $4x^2-3y^2=13$.

28. $x^2+y^2=62-x-y$; $xy=14$.

29. $2x^2-3xy+4y^2=6$; $x^2+3y^2=7$.

30. $\frac{1}{x}+\frac{1}{y}=\frac{5}{3}$; $xy=4$.

31. $x^4+y^4=626$; $x+y=6$.

32. $2x^2+xy-3y^2=8$; $x^2-2y^2=7$.

33. $3x^2-5y^2-3x-2y=9$; $2x-3y=1$.

34. $x^5+y^5=1056$; $x+y=6$.

35. $x^{-1}-y^{-1}=1$; $x^{-3}-y^{-3}=3\frac{1}{4}$.

36. $x+xy=2$; $y+xy=4\frac{1}{2}$.

37. $3x^2+2xy-2y^2=6$; $2x^2+xy-3y^2=3$.

38. $x^2-y^2=16$; $x^2+2xy=4-y^2$.

39. $xy=1\frac{1}{2}$; $2x-5y=2$.

40. $x^2+y^2+x+y=20$; $x+y=3$.

41. $\frac{1}{x-y}-\frac{1}{x+y}=-4\frac{4}{5}$; $\frac{3}{x}+\frac{4}{y}=\frac{1}{xy}$.

42. $\frac{x}{y}-\frac{y}{x}=-2\frac{2}{3}$; $2x+3y=2$.

43. $2x+y+2xy=5$; $x+3y+2xy=1$.

44. $2x+3y=10$; $2x^2y^2+45xy=72$.

45. $x^2+xy+y^2=7$; $x^2-xy+y^2=19$.

46. $5x^2-y^2=11$; $3xy+y^2=-9$.

47. $x^2y^2+14xy+24=0$; $3x+y=5$.

48. $x+y=4$; $x^4+y^4=82$.

49. $x^3+y^3=37$; $x^2y+xy^2=-12$.

50. $x^2+y^2-5x+5y=30$; $xy=8$.

51. $x^2+xy+y^2=19$; $x^4+x^2y^2+y^4=931$.

52. $\frac{1}{x}-\frac{1}{y}=4$; $\frac{1}{x^2}+\frac{1}{y^2}=8\frac{1}{2}$.

53. $x-\sqrt{xy}+y=9$; $x^2+xy+y^2=189$.

54. $x^2+2xy+3y^2=a^2+2b^2$; $x+y=a$.

55. $x^2+y^2+x-y=32$; $xy=10$.

56. $x^{\frac{1}{2}}+y^{\frac{1}{2}}=5$; $x^{\frac{1}{2}}y^{\frac{1}{2}}=6$.

57. $x+y=117$; $\sqrt[3]{x}+\sqrt[3]{y}=3$.

58. $x^{\frac{1}{3}}-2y^{\frac{1}{3}}=1$; $x-8y=19$.

59. $x^2+xy+y^2=7$; $x^4+x^2y^2+y^4=91$.

60. $x+y=3$; $x^5+y^5=33$.

61. $x^2+4y^2-x-2y-42=0$; $xy=12$.

62. $x^3-y^3=117$; $x^2y-xy^2=30$.

63. $x-y=2$; $\sqrt{x}+\sqrt{y}=2$.

64. $x+y=13+\sqrt{xy}$; $x^2+y^2=273-xy$.

65. $\sqrt{\frac{x}{y}}+2\sqrt{\frac{y}{x}}=3;\ x+y=5.$

66. $2x+\sqrt{xy}=12;\ y+\sqrt{xy}=18.$

67. $x^2+xy+y^2=x^2-xy+y^2=1.$

68. $x^2+y^2=xy=1.$ 69. $x^2-y^2=xy=x+y.$

70. $x^2+y^2=3xy-1;\ x^4+y^4=17.$

71. $\frac{x}{2a}+\frac{y}{b}=-2b;\ \frac{4ab}{x}+\frac{3b^2}{y}=1.$

72. $(x-2y)^2-x+2y=6;\ 3x-5y=11.$

73. $xy+x+y=7;\ xy(x+y)=12.$

74. $2x^2+3y^2=8;\ 2(x-1)^2+3(y+1)^2=5.$

75. $x^2+y^2=xy+19;\ x+y=xy-7.$

PROPERTIES OF QUADRATICS

Exercise 55

Form the quadratic equations which will have the following roots:

1. 7, 5.
2. 2, $3\frac{1}{2}$.
3. 6, -4.
4. 4, $-1\frac{1}{2}$.
5. $\frac{1}{3}$, $\frac{1}{2}$.
6. -1, $\frac{1}{2}$.
7. 0, 5.
8. $-2\frac{1}{3}$, 0.
9. a, -1.
10. 7, -7.
11. $\sqrt{5}$, $-\sqrt{5}$.
12. $\sqrt{-3}$, $-\sqrt{-3}$.
13. a, $a-1$.
14. $3+a$, $-3-2a$.
15. $2a$, $\frac{1-3a}{2}$.
16. ± 10.
17. $\pm\sqrt{a-1}$.
18. $1+\sqrt{2}$, $1-\sqrt{2}$.

19. $3 \pm \sqrt{2}$.

20. $\pm \sqrt{11} - 5$.

21. $\frac{7 \pm \sqrt{70}}{2}$.

22. $\frac{3 \pm \sqrt{3}}{3}$.

23. $\frac{-7 \pm \sqrt{5}}{2}$.

24. $\frac{a \pm \sqrt{a^2 - 1}}{b}$.

25. $5 \pm \sqrt{-1}$.

26. $\frac{3 \pm \sqrt{-1}}{2}$.

27. $\frac{5 \pm \sqrt{-2}}{3}$.

28. $\frac{-7 \pm 2\sqrt{-1}}{5}$.

29. $\frac{a \pm \sqrt{-b}}{2}$.

30. $\frac{5 \pm 3\sqrt{-5}}{4}$.

Without finding the actual values of x, tell what the sum of the roots is; their product; their character:

31. $x^2 - 5x - 24 = 0$.

32. $x^2 + 5x - 1 = 0$.

33. $2x^2 - 3x + 1 = 0$.

34. $3x^2 + x - 10 = 0$.

35. $x^2 - 3x + 5 = 0$.

36. $5x^2 - 6x + 2 = 0$.

37. $4x^2 + 4x + 1 = 0$.

38. $4x^2 = -x + 4$.

39. $9x^2 + 1 = 6x$.

40. $12x^2 + 7x = -6$.

41. $4x^2 - 3x = 0$.

42. $4x^2 = 7$.

43. $x^2 - x = 1$.

44. $3x^2 + 5x + 3 = 0$.

45. $25x^2 = 10x - 1$.

46. $3x^2 + 5x = 0$.

47. $3x^2 + 5 = 0$.

48. $2x^2 - x = 1$.

49. $16x^2 - 40x = -25$.

50. $7x^2 + 13x = 5$.

Find the values of k which will make the following equations have equal roots:

51. $2x^2 - 2x + k = 0$.

52. $kx^2 - 4x + 3 = 0$.

53. $x^2 + x = -k$.

54. $kx^2 = 3x - 2$.

55. $3x^2 + 2x = 1 - k$.

56. $kx^2 - kx + 1 = 0$.

57. $5x^2 = 4x - 2k + 1$.

58. $x^2 - kx + 9 = 0$.

59. $kx^2 + kx = -8x - 9$.

60. $x^2 + 49 = kx + 3x$.

61. $4x^2 = kx - k - 5$.

62. $11x^2 + 1 = 3x - kx^2 + kx$.

63. $kx^2 - kx = 7x^2 + 9x - 25$.

64. $3kx^2 + 6k = 5x(k+3) - 7$.

Resolve into factors:

65. $x^2 - 3x + 1$.

66. $x^2 - x - 3$.

67. $3x^2 - 2x - 2$.

68. $x^2 + 4$.

69. $x^2 + x + 1$.

70. $x^2 - 7xy - y^2$.

71. $a^2 - 5ab + b^2$.

72. $17 - 8x + x^2$.

73. $5x^2 + 8x - 2$.

74. Explain the rules for determining whether the roots of an equation are real or imaginary. Equal or unequal. Rational or irrational.

75. If the sum of the roots of a quadratic is 3 and their product is 2, find the difference of the roots. Find the difference of the squares of the roots. Find the sum of the reciprocals of the roots.

76. Find the condition that one root of $ax^2 + bx + c = 0$ shall be the reciprocal of the other. Find the condition that one root shall be double the other. One three times the other.

77. If m and n stand for the roots of $2x^2 + 5x - 3 = 0$, find the values of: (*a*) $m + n$. (*c*) $m - n$. (*e*) $\frac{1}{m} + \frac{1}{n}$.

(*b*) mn. (*d*) $m^2 - n^2$. (*f*) $m^2 + n^2$.

78. Find the values of the same expressions in the equation $3x^2 = 13x + 10$. Also in equation $3x^2 - x + 1 = 0$.

79. Form the quadratic whose roots shall be $\frac{2}{3}$ and $\frac{5}{6}$. Form that whose roots shall be $\frac{3}{2}$ and $\frac{6}{5}$. Compare the results.

RATIO AND PROPORTION

Exercise 56

1. Find a mean proportional between $5\frac{1}{3}$ and 27. Between m and n^2. Between $\frac{x^2-y^2}{a}$ and $\frac{x+y}{a}$.

2. Find a fourth proportional to 3, 5, 12. To a, $a+1$, a^2. To 6, 8, $10\frac{1}{2}$. To 8, $10\frac{1}{2}$, 6.

3. Find a third proportional to 4 and 10. To 3 and $3\frac{3}{4}$. To a and $x-1$. To $\frac{1}{2}$ and $\frac{2}{3}$.

4. Solve $2x-1 : 3x-2 = 3(x+1) : 5x+1$.

5. Solve $x-5 : 3x+1 = 5-8x : 3(1-2x)$.

6. Solve $\frac{1}{a} : \frac{1}{b} = \frac{1}{a-b} : \frac{1}{x^2}$.

7. Solve $\frac{a^2-b^2}{c(a+c)} : \frac{a+b}{a^2-c^2} = x : a+\frac{ac}{a-c}$.

8. Solve

$$2x^2-3x+1 : 3x^2-3x+1 = 3x^2-2x-5 : 4x^2-2x-5.$$

9. Solve $\frac{2x^2-4x-1}{2x^2+2x-1} = \frac{x^2+x-2}{x^2+13x-2}$.

10. Solve $\frac{x^2+3x-7}{x^2-5x+6} = \frac{x^2+4x+10}{x^2-4x+4}$.

11. Solve $\frac{4x^3-3x-6}{4x^3+x-6} = \frac{x^2+x+2}{x^2+x+3}$.

12. Solve $\frac{x^3-2x^2+2}{x^3-3x^2+2} = \frac{2x^2+x-1}{2x^2-x-1}$.

13. Solve $\sqrt{x+4} : \sqrt{x-1} = \sqrt{6x+6} : \sqrt{5x-9}$.

14. Solve $\sqrt{3x-2} : \sqrt{4x+1} = \sqrt{7x+2} : 2\sqrt{5x-1}$.

15. Two numbers are in the ratio of 3 : 7 and their sum is 60. Find them.

16. Three numbers are in the ratio of 2 : 3 : 4 and their sum is 63. Find them.

17. Find two numbers in the ratio of 2 : 5, the sum of whose squares is 464.

18. Find three numbers in the ratio of 1 : 2 : 3, the sum of whose squares is 126.

19. What number added to each of the numbers 2, 5, 11, 15 will make the sums proportional?

20. Find a mean proportional and a third proportional to 5 and 20. Also to $3\frac{1}{3}$ and $7\frac{1}{2}$.

21. If $a : b = c : d$, prove the property of "composition" by use of the equivalents, $a = bx$ and $c = dx$. Prove "division" by the same method.

22. If $a : b = c : d = e : f = g : h$, prove by the method of example 21 that $a + c + e + g : b + d + f + h = a : b = c : d = \text{etc.}$

23. If $a : b = c : d$, prove that $a + 3c : b + 3d = 2a + c : 2b + d$.

24. If $m : n = p : q$, prove that $m + n : p + q = m - 2n : p - 2q$.

25. If $x : y = z : w$, prove that

$$x^2 + y^2 : z^2 + w^2 = (x - ny)^2 : (z - nw)^2.$$

26. If $p : q = r : s$, prove that

$$\sqrt{p^2 + r^2} : \sqrt{q^2 + s^2} = ap - br : aq - bs = p : q.$$

27. If $a : b :: b : c$, prove that $a + 3b : b + 3c = a : b$ by use of the equivalents $a = cx^2$ and $b = cx$.

28. If y is a mean proportional between x and z, prove that

$$x - 2y : y - 2z = 2x - 3y : 2y - 3z.$$

29. If a, b, c, d are in *continued* proportion, prove by use of the equivalents $a = dx^3$, $b = dx^2$, $c = dx$ that $a + b + c : a + b = b + c + d : b + c$.

30. If a, b, c, d are in continued proportion, prove that

$$a + b^2 : c + c^2 = b + c^2 : d + d^2.$$

31. If a, b, c are in continued proportion, prove that

$$a + b : b + c = b^3 : ac^2.$$

32. If a, b, c, d are in continued proportion, prove that

$$a^2 + b^2 + c^2 : b^2 + c^2 + d^2 = (a + c)(a - c) : (b + d)(b - d) = a^2 : b^2.$$

33. If $\frac{x}{b+c} = \frac{y}{a+c} = \frac{z}{a-b}$, prove that $x - y + z = 0$.

34. If $\frac{l}{b-c} = \frac{m}{c-a} = \frac{n}{a-b}$, prove that $l + m + n = 0$.

35. If $a + 2b + c : b + c = a + b : b$, prove that b is a mean proportional between a and c.

36. Find two numbers in the ratio of $2 : 3$ such that the sum of their squares is to their product increased by 2, as $2 : 1$.

37. If 1 be added to each of two numbers, their ratio is $1 : 2$. The difference of their squares is to 3 more than their product as $5 : 3$. Find them.

38. There are two numbers such that the ratio of the sum of their cubes and the cube of their sum is $7 : 1$; and if 6 be added to each, the ratio of these sums is $1 : 4$. Find them.

39. For what value of x will $2x - 1$ be a mean proportional between $x + 5$ and $4x - 13$?

40. What values must x have in order that $2x - 7$, $3x + 1$, $4x - 3$, $5(x + 1)$ may form a true proportion?

VARIATION

Exercise 57

1. If x varies as y and $y = 2$ when $x = 12$, find x when $y = \frac{1}{3}$.

2. If $x \propto y^2$ and $x = \frac{1}{2}$ when $y = \frac{1}{2}$, find y when $x = 18$.

3. If A varies inversely as B and $A = -6$ when $B = -\frac{1}{2}$, find A when $B = \frac{2}{3}$.

4. If A varies jointly as B and C and $A = 9$ when $B = C = 6$, find A when $B = 5$ and $C = -8$.

5. If x varies directly as y and inversely as z, and $x = 2$ when $y = 3$ and $z = 6$, find y when $x = 8$ and $z = -3$.

6. If $x \propto y$ and x is 3 when $y = \frac{3}{4}$, find an equation between x and y.

7. If $x \propto \frac{1}{y}$ and $y = -5$ when $x = 2$, find the equation joining x and y.

8. If $x \propto \frac{y}{z}$ and $x = 15$ when $y = 5$ and $z = 4$, find x in terms of y when z is -1.

9. If $x \propto (2y + 5)$ and $x = 3$ when $y = -2$, find y if $x = 6$.

10. Given that $y^2 \propto (x^2 + 1)$ and $x = 7$, when $y = 10$, find x when $y = \sqrt{10}$.

11. If u is equal to the sum of two quantities, one of which varies as x and the other inversely as x, and if $u = -1$ when $x = \frac{2}{3}$, and $u = 1$ when $x = 1$, find the equation between u and x.

12. If v is equal to the sum of two quantities, one of which varies as x^2 and the other inversely as y, and $v = -1$, when $x = \frac{1}{2}$, $y = 2$; and $v = 7$ when $x = 2$, $y = 3$; find the equation for v in terms of x when $y = -1$.

13. Given that $y =$ the sum of three quantities which vary as x, x^2, and x^3 respectively. When $x=1$, $y=4$; when $x=2$, $y=8$; when $x=3$, $y=18$. Express y in terms of x.

14. If y varies inversely as x^2-1 and $y=-5$ when $x=4$, find x when $y=-15$.

15. If y varies inversely as $(2x+1)(x-3)$ and $y=-\frac{1}{2}$ when $x=2$, find x when $y=1\frac{1}{3}$.

16. If the area of a circle varies as the square of its radius, and the area of a circle whose radius is 7 is 154, find the area of the circle whose radius is 10.

17. Find the radius of the circle equivalent to the sum of two circles whose radii are 5 and 12 respectively.

18. The pressure of the wind upon a plane surface varies jointly as the area of the surface and the square of the wind's velocity. The pressure on a square foot is 1 pound when the wind is blowing at the rate of 15 miles per hour. Find the velocity of the wind when the pressure on a square yard is 36 pounds.

19. If w varies as the sum of x, y, and z, and $w=3$ when $x=3$, $y=-4$, $z=6$, find x if $w=-3$, $y=3\frac{1}{2}$, $z=-9$.

20. If w is equal to the sum of two quantities, one of which varies as x, and the other jointly as y and z, and $w=-3$ when $x=2$, $y=6$, $z=-1$; and $w=-2$ when $x=4$, $y=2$, $z=-3$; find the equation combining the four quantities, w, x, y, and z.

21. If the square of x varies as the cube of y, and $x=3$ when $y=2$, find y when $x=24$.

22. The area of a triangle varies jointly as its base and altitude. Find the altitude of a triangle whose base is 23, equivalent to the sum of two triangles whose bases are 15 and 22 and whose altitudes are 10 and 12 respectively.

ARITHMETICAL PROGRESSION

Exercise 58

In the following 16 examples tell what a is, what d is, what n is. Also find l and s in each.

1. 5, 7, 9, ···, to 15 terms.
2. 6, 9, 12, ···, to 10 terms.
3. $-2, -3\frac{1}{2}, -5, \cdots$, to 45 terms.
4. 3, 3.1, 3.2, ···, to 300 terms.
5. 8, 7.5, 7, ···, to 60 terms.
6. $2\frac{2}{3}, 2\frac{1}{2}, 2\frac{1}{3}, \cdots$, to 55 terms.
7. $-3\frac{1}{3}, -2\frac{5}{6}, -2\frac{1}{3}, \cdots$, to 75 terms.
8. $1+x, 1+3x, 1+5x, \cdots$, to 10 terms.
9. Odd numbers to 37 terms.
10. Numbers divisible by 7 to 15 terms.
11. Numbers divisible by 3 to 20 terms.
12. 5, 10, 15, ···, to r terms.
13. 1, 2, 3, 4, ···, to x terms.
14. 2, 6, 10, 14, ···, to n terms.
15. The first n odd numbers.
16. The first $2n$ even numbers.

Insert, between

17. 11 and 32, 5 arithmetical means.
18. $7\frac{1}{2}$ and 30, 9 arithmetical means.

19. $38\frac{2}{3}$ and $-44\frac{2}{3}$, 99 arithmetical means.

20. 17 and 3, 12 arithmetical means.

Find d and s if:

21. $a=5,\ l=25,\ n=11.$

22. $a=-13,\ l=26,\ n=14.$

23. $a=1\frac{1}{2},\ l=36,\ n=24.$

24. $a=12\frac{1}{3},\ l=-13\frac{2}{3},\ n=40.$

Find n and s if:

25. $a=6,\ d=2,\ l=80.$

26. $a=-17,\ d=4,\ l=39.$

27. $a=3\frac{1}{4},\ d=\frac{1}{4},\ l=10\frac{1}{2}.$

28. $a=9\frac{1}{6},\ d=-\frac{1}{3},\ l=-15\frac{1}{2}.$

Find a and s if:

29. $d=3,\ l=38,\ n=11.$

30. $d=1\frac{2}{3},\ l=69,\ n=41.$

31. $d=-2,\ l=-25,\ n=27.$

32. $d=-\frac{2}{3},\ l=6\frac{1}{6},\ n=20.$

Find l and d if:

33. $a=5,\ n=9,\ s=297.$

34. $a=3\frac{1}{2},\ n=15,\ s=78\frac{3}{4}.$

35. $a=-1\frac{2}{3},\ n=30,\ s=530.$

Find n and d if:

36. $a=8,\ l=41,\ s=294.$

37. $a=3\frac{1}{2},\ l=42\frac{1}{2},\ s=621.$

38. $a=8,\ l=0,\ s=100.$

39. $a=-3\frac{1}{2},\ l=-36,\ s=-790.$

Find a and l, if:

40. $d=3,\ n=13,\ s=260.$

41. $d=\frac{1}{4},\ n=20,\ s=102\frac{1}{2}.$

42. $d=-\frac{3}{4},\ n=35,\ s=-306\frac{1}{4}.$

Find a and d, if:

43. $l=47,\ n=23,\ s=575.$

44. $l=11\frac{2}{3},\ n=37,\ s=209\frac{2}{3}.$

45. $l=-16\frac{1}{2},\ n=43,\ s=43.$

Find n and l, if:

46. $a=3,\ d=2,\ s=80.$ 47. $a=2,\ d=-3,\ s=-328.$

48. $a=5,\ d=-\frac{1}{2},\ s=27.$

Find n and a, if:

49. $d=5,\ l=32,\ s=119.$ 51. $d=\frac{1}{3},\ l=6,\ s=45.$

50. $d=-\frac{1}{2},\ l=5\frac{1}{2},\ s=25.$ 52. $d=-\frac{2}{3},\ l=-3,\ s=13.$

53. How many numbers are there between 100 and 1000 that are exactly divisible by 7? Find their sum.

54. Find the sum of all the numbers of two figures each that are divisible by 8.

55. Find the sum of the first 50 odd numbers.

56. In the series 2, 5, 8, ···, which term is 98?

57. How many terms must be taken from the series 3, 5, 7, ···, to make a total of 255?

58. Which term of the series $1\frac{1}{2}$, 2, $2\frac{1}{2}$, ···, is 24? How many consecutive terms must be taken from this series to make 84?

59. The 7th term of an A. P. is 17, and the 12th term is 27. Find the 1st term. The 3d term.

60. The 10th term of an A. P. is $\frac{2}{5}$, and the 18th is $3\frac{3}{5}$. Find the 1st term. The 100th term. Sum of 20 terms.

61. How is a single arithmetical mean between 2 numbers found most readily? How do you determine whether or not 3 numbers are in A. P.?

62. Find x, so that $3-5x$, $1+2x$, $4+7x$, shall form an A. P.

63. The sum of 4 numbers in A. P. is 46, and the product of the 2d and 3d is 130. Find them.

64. The sum of 3 numbers in A. P. is 27, and the sum of their squares is 275. Find them.

65. A body freely falling from a position of rest will fall $16\frac{1}{12}$ feet the first second, $48\frac{1}{4}$ feet the second second, $80\frac{5}{12}$ feet the third, and so on. Find the distance fallen during the 10th second. How far in 10 seconds? How far in 20th second? How far in 20 seconds?

66. Find x, so that $3+2x, 5+6x, 9+5x$, shall form an A. P.

67. Which term of the series $2\frac{1}{2}, 3\frac{3}{4}, 5, \cdots$, is 45?

68. How many consecutive terms in the series $2\frac{1}{2}, 3\frac{3}{4}, 5, \cdots$, will make $67\frac{1}{2}$? Interpret the negative result.

69. If the 6th term of an A. P. is 9 and the 16th term is $22\frac{1}{3}$, find the 25th term and the sum of 30 terms.

70. Find the sum of the series $x, 3x, 5x, 7x, \cdots$, to x terms.

71. Find the sum of all the numbers between 100 and 600 that are divisible by 11.

72. Find x, so that $2x-1, 3x+2, 6x+8$, shall be an A. P.

73. What will x and y each be, if the four terms $2x-y$, $x+2y, 3x+y, 7x-10$, form an A. P.?

74. Find the sum of 15 terms of an A. P. of which the middle one is $10\frac{1}{3}$.

75. Find the sum of $\frac{n+1}{n}+\frac{n+2}{n}+\frac{n+3}{n}\cdots$ to n terms.

76. A boy travels at the rate of 1 mile the first day, 2 the second, 3 the third, and so on; 6 days later a man sets out from the same place to overtake him, traveling 15 miles every day. How many days must elapse after the second starts before they are together? Interpret both results.

77. The sum of n terms of the series 21, 18, 15, $\cdots$, is equal to the sum of the same number of terms of the series 3, $3\frac{3}{11}$, $3\frac{6}{11}$, $\cdots$. Find n.

78. Find the sum of 41 terms of an A. P. whose 21st term is 100.

GEOMETRICAL PROGRESSION

Exercise 59

Find l and s in each:

1. 3, 6, 12, ···, to 8 terms.
2. 2, 8, 32, ···, to 5 terms.
3. 40, 20, 10, ···, to 6 terms.
4. 2.1, 21, 210, ···, to 5 terms.
5. 54, 18, 6, ···, to 5 terms.
6. 3.2, 0.32, .032, ···, to 6 terms.
7. $3, \frac{6}{7}, \frac{12}{49}, \cdots$, to 5 terms.
8. $\frac{8}{9}, \frac{4}{3}, 2, \cdots$, to 7 terms.
9. $1\frac{1}{2}, -3, 6, \cdots$, to 9 terms.
10. $-5, 15, -45, \cdots$, to 5 terms.
11. $3\frac{1}{5}, 1\frac{3}{5}, \frac{4}{5}, \cdots$, to 10 terms.
12. $16\frac{7}{8}, -11\frac{1}{4}, 7\frac{1}{2}, \cdots$, to 5 terms.
13. $1 + x + x^2 + x^3 \cdots$, to b terms.
14. $32 - 16 + 8 - 4 + 2 - 1 \cdots$, to n terms.

Find r and s, if:

15. $a = 3, \quad l = 48, \quad n = 5.$
16. $a = 5, \quad l = 405, \quad n = 5.$
17. $a = 13\frac{1}{2}, \quad l = 1\frac{7}{9}, \quad n = 6.$

Find a and s, if:

18. $l = \frac{1}{4}, \quad n = 6, \quad r = \frac{1}{2}.$
19. $l = 85\frac{1}{3}, \quad n = 5, \quad r = 1\frac{1}{3}.$
20. $l = \frac{1}{9}, \quad n = 5, \quad r = -\frac{2}{3}.$

Find n and s, if:

21. $a=5,\ l=160,\ r=2.$ 23. $a=24,\ l=\frac{3}{8},\ r=\frac{1}{2}.$

22. $a=3,\ l=1875,\ r=5.$ 24. $a=\frac{3}{4},\ l=-24,\ r=-2.$

Find r and n, if:

25. $a=2,\ l=486,\ s=728.$ 27. $a=1\frac{2}{3},\ l=135,\ s=201\frac{2}{3}.$

26. $a=56,\ l=1\frac{3}{4},\ s=110\frac{1}{4}.$ 28. $a=\frac{8}{9},\ l=-\frac{243}{16},\ s=-8\frac{109}{144}.$

Insert, between

29. 4 and 972, 4 geometrical means.

30. 7 and 896, 6 geometrical means.

31. $5\frac{1}{8}$ and $40\frac{1}{2}$, 4 geometrical means.

32. 20 and $-\frac{5}{128}$, 8 geometrical means.

33. $7\frac{1}{2}$ and $\frac{80}{81}$, 4 geometrical means.

Find the sum of each series to infinity:

34. $6, 3, 1\frac{1}{2}, \cdots.$ 38. $8\frac{8}{9}, -6\frac{2}{3}, 5, \cdots.$

35. $1, -\frac{1}{2}, \frac{1}{4}, \cdots.$ 39. 8.3, 0.83, .083, $\cdots.$

36. $15, 5, 1\frac{2}{3}, \cdots.$ 40. .72, .0072, .000072, $\cdots.$

37. 18, 12, 8, $\cdots.$ 41. $1\frac{1}{8}$, 0.75, 0.5 $\cdots.$

42. 0.4545, $\cdots.$ 44. 3.8181, $\cdots.$ 46. 2.34848, $\cdots.$

43. 0.05454, $\cdots.$ 45. 5.12727, $\cdots.$ 47. 1.026363, $\cdots.$

48. If the 3d term of a G. P. is 36 and the 6th term is 972, find the 1st and 2d terms.

49. If the 4th term is 24 and the 8th term is 384, find the first 2 terms.

50. The 3d term is 4 and the 7th is $20\frac{1}{4}$. Find the first 2 terms.

51. In the G. P. 2, 6, 18, ···, which term is 486?

52. How many terms must be taken from the series 9, 18, 36, ···, to make a sum of 567?

53. How many consecutive terms in the series 48, 24, 12, ···, are required to make $95\frac{1}{4}$?

54. The 1st term of a G. P. is 8. Its sum to infinity is 32. Find the ratio.

55. How can a single geometric mean be determined most readily? How does one test a series to determine whether it is a G. P. or not?

56. Find x, if $2x-4$, $5x-7$, $10x+4$, are in G. P.

57. There are 3 numbers in A. P. whose common difference is 4. If 2, 3, 9, be added to them respectively, the sums form a G. P. Find the numbers.

58. The sum of a G. P. to infinity is 18 and the 2d term is 4. Find the 1st term and ratio.

59. If the series $x+1$, $x+3$, $4x-3$, is geometric, find x. Find x if it is an A. P. Find the 4th term of the series in each case.

60. Tell whether each of the following series is arithmetical or geometrical:

(*a*) 3, 6, 12, ···. (*c*) 12, 18, 25, ···.

(*b*) 6, 12, 18, ···. (*d*) $3\frac{3}{4}$, $1\frac{1}{2}$, 0.6, ···.

61. The sum of three numbers in G. P. is 65. The sum of the first two is $\frac{1}{8}$ the sum of the last two. Find them.

62. Divide 49 into 3 parts in G. P. such that the sum of the 1st and 3d parts is $2\frac{1}{2}$ times the middle part.

63. The sum of 3 numbers in G. P. is 14 and the sum of their reciprocals is $\frac{7}{8}$. Find them.

64. Insert between 6 and 16 two numbers, such that the first three of the four shall be in A. P. and the last three in G. P.

65. If the series $3\frac{1}{3}, 2\frac{1}{2}, \cdots$, be an A. P., find the 105th term. If a G. P., find the sum to infinity.

66. The sum of $ 240 was divided among 4 men in such a way that the shares were in G. P., and the difference between the greatest and least shares is to the difference between the other two, as 13 : 3. Find each share.

67. What number added to each of the numbers 2, 5, 11, will make sums that are in G. P. ?

68. Find x, so that $5+x$, $5-x$, $2(1-5x)$, shall be in G. P.

69. If $4x-1, 6x+1, 5(2x+1)$, are in G. P., find x and find the ratio. Also find the next term.

70. If the first term of a G. P. is 6 and the sum to infinity is 18, find the third term.

71. If a man ascends a mountain at the rate of 81 yards the first hour, 54 yards the second, 36 yards the third, etc., how many hours will he require to ascend 211 yards ?

72. There are 4 numbers, the first three of which are in G. P., and the last three are in A. P. The sum of the first and last is 14, and the sum of the second and third is 12. Find the numbers.

73. A ball thrown vertically into the air 150 feet falls and rebounds 60 feet. It falls again and rebounds 24 feet, and so on until it comes to rest on the ground. Find the entire distance through which the ball has traveled.

74. Prove that equimultiples of a G. P. are also in G. P., and that alternate terms of a G. P. form another G. P.

PERMUTATIONS AND COMBINATIONS

Exercise 60

1. How many numbers of 5 different figures each can be formed from our 9 significant digits?

2. How many words of 4 letters each can be formed from the 26 letters in our alphabet, no letter being repeated in the same word?

3. Find the number of committees, each containing 5 boys, that can be selected from a room of 20 boys.

4. Find the number of combinations of 8 objects each that can be formed from 25 objects.

5. How many different words can be formed from the letters in the word *Thursday*, using all its letters each time?

6. From the members of a party of 30 people, a board of 4 officers is to be chosen. In how many ways can this occur?

7. From the letters in the word *Republican* how many words of 4 letters can be found? Of 5 letters? Of 7 letters?

8. The prime factors of a certain number are 2, 5, 7, 11, and 17. How many exact divisors (except itself and unity) has this number?

9. It is required to place 20 dissimilar bouquets in the form of an arch. In how many ways can they be arranged?

10. From the 9 significant digits, how many numbers can be formed each containing 1 digit? Two different digits? 3? 4? 5? 6? 7? 8? 9? All together?

11. There are 25 points in a certain plane; these are joined so as to form triangles having the vertices at the points. How many triangles will there be?

12. From the letters in the word *handiwork* how many words of 5 letters can be formed? How many of these will contain the h? the w? How many will begin with d? How many will contain both d and i? How many will not contain d?

13. A man has 5 pairs of trousers, 8 vests, and 6 coats. In how many different costumes can he appear?

14. Six persons enter a car in which there are 10 seats. In how many ways can they be seated?

15. In how many ways can a baseball nine be arranged provided the pitcher is always the same? Provided the pitcher and catcher are always the same individuals?

16. In how many ways can 10 people arrange themselves around a circular table?

17. How many words can be formed from the letters in the word *latin*, the 2d and 4th being always vowels?

18. How many words can be formed from the letters in the word *united*, the even places being always occupied by consonants?

19. How many words can be formed from the letters in the word *education*, provided the 2d, 4th, 6th, and last letters are always consonants?

20. From our 9 digits how many numbers can be formed, each containing 6 figures? How many of these will contain the figure 5? How many will not contain a 7? How many will contain both 5 and 7? How many will begin with 3? End with 4? How many will be odd?

21. From 10 gentlemen and 8 ladies how many committees can be chosen, each containing 3 gentlemen and 2 ladies?

22. From 10 consonants and 5 vowels how many words, each containing 4 consonants and 3 vowels, can be formed?

23. There are 8 Democrats and 10 Republicans belonging to a certain board. How many committees can be chosen each having 4 Democrats and 5 Republicans?

24. Out of 4 vowels and 9 consonants there are words to be constructed, each consisting of 2 vowels and 6 consonants. How many can there be?

25. From 6 white balls, 4 red balls, and 8 black balls, how many combinations can be made each to contain 3 white, 2 red, and 4 black balls?

26. From 4 labials, 6 vowels, 5 palatals, how many words can be made each consisting of 2 labials, 3 vowels, and 2 palatals?

27. How many different sums of money can be made from the following coins: cent, 5-cent, dime, quarter, half dollar, and dollar?

28. A guard of 5 men must be selected every night out of a detachment of 32 men. For how many nights can a different guard be selected? How many times will each soldier serve?

29. A company of 15 merchants, 12 lawyers, and 8 teachers wish to form a commission from their number, consisting of 4 merchants, 3 lawyers, and 2 teachers. How many ways are there in which they can do it?

30. Find the number of permutations that can be made from the letters in the following words using all the letters:

(*a*) *Recess.* (*c*) *Bumblebee.* (*e*) *Concurrence.*
(*b*) *Possess.* (*d*) *Tennessee.* (*f*) *Unostentatious.*

31. In how many different ways can one mail 4 letters in a village containing 7 letter boxes?

32. How many different quantities can be weighed with the following weights: 1 ounce, 3 ounces, 8 ounces, 10 ounces, 1 pound, 5 pounds, and 10 pounds?

33. With 2 violet, 2 indigo, 3 blue, 4 green, 1 yellow, 1 orange, and 2 red flags, how many signals can be made if all the flags are used and always kept in a vertical column?

34. From 7 consonants and 5 vowels how many words can be formed, each consisting of 4 consonants and 2 vowels?

35. A plane is determined by 3 points, if they are not all in a straight line. How many planes are determined by 100 points (no four of them lying in the same plane)?

BINOMIAL THEOREM

Exercise 61

Expand:

1. $(a-2)^5$.
2. $(2a+1)^4$.
3. $(1-3a^2x)^3$.
4. $(1-y)^7$.
5. $(2m+n^2)^6$.
6. $(3a-\sqrt{ab})^4$.
7. $(a\sqrt{x}-1)^5$.
8. $(\sqrt{2}-\sqrt{3})^4$.
9. $(\sqrt[3]{6}-2)^5$.
10. $(\sqrt{2}+2\sqrt{6})^3$.
11. $(\sqrt{a}-\sqrt{n})^{10}$.
12. $(a^2-\frac{1}{2}a\sqrt[3]{c})^5$.
13. $(1-3\sqrt{3})^4$.
14. $(\sqrt{\frac{1}{2}}-3\sqrt{2a})^4$.
15. $(\sqrt{3x}-\sqrt[3]{2xy})^5$.
16. $(2-\sqrt{-1})^3$.
17. $(3\sqrt{-1}+2\sqrt{5})^4$.
18. $(ax^{-1}-\sqrt{a^{-1}x})^6$.
19. $\left(\sqrt{\frac{1}{3a}}-3\sqrt{-3a}\right)^3$.
20. $\left(\frac{2a}{\sqrt{n}}+\frac{1}{an}\right)^5$.
21. $(a\sqrt[3]{x^{-2}}-x\sqrt{a^{-1}})^6$.
22. $(2a\sqrt{2x}+\sqrt[3]{4})^4$.
23. $(\sqrt{2a}-\sqrt[3]{-3a^2})^3$.
24. $\left\{\frac{2\sqrt{a}}{\sqrt[4]{n^3}}+\frac{3\sqrt{2n}}{\sqrt[3]{a}}\right\}^6$.
25. $\left\{\frac{b^{-1}\sqrt{c}}{a\sqrt{a}}-\frac{2\sqrt{2a}}{\sqrt[3]{c}}\right\}^5$.

Find only the term required:

26. The 7th term of $(x+\sqrt{y})^{11}$.

27. The 5th term of $(1-2x)^{9}$.

28. The 4th term of $(a+3\sqrt{x})^{10}$.

29. The 6th term of $(2n\sqrt[3]{m}-1)^{11}$.

30. The 8th term of $(a\sqrt{x}-x\sqrt{a})^{12}$.

31. The 7th term of $(\frac{1}{2}-2x\sqrt[3]{y})^{11}$.

32. The 5th term of $(\sqrt{ax}+\sqrt[4]{x^3})^{13}$.

33. The 4th term of $(\sqrt{6}-\sqrt{3})^{6}$.

34. The 5th term of $(\sqrt{3}+\sqrt{-2})^{9}$.

35. The 7th term of $(\sqrt{2}-a\sqrt{3x})^{10}$.

36. The 9th term of $(\frac{1}{12}\sqrt[3]{a}+2\sqrt{3ab})^{11}$.

37. The 4th term of $(\sqrt{2}-5\sqrt{-1})^{7}$.

38. The 5th term of $\left[\frac{\sqrt{x}}{\sqrt[3]{y^2}}+\frac{\sqrt[3]{y}}{x\sqrt{x}}\right]^{16}$.

39. The term containing x^9 in $(1-x\sqrt{x})^{10}$.

40. The term containing x^7 in $(x+2\sqrt{x})^{9}$.

41. The term containing $x^{\frac{1}{3}}$ in $(\sqrt{x}-\sqrt[3]{xy})^{11}$.

42. The term containing $x^{\frac{7}{2}}$ in $(2\sqrt[3]{x}-\sqrt{2x})^{8}$.

43. The term containing x^{-4} in $\left(\frac{1}{6x}-\frac{3}{\sqrt{x^{-1}}}\right)^{13}$.

Expand to four terms:

44. $(a+b)^{-3}$.

45. $(a-x)^{-1}$.

46. $(x-2)^{-5}$.

47. $(a+y)^{\frac{1}{2}}$.

48. $(a^2-4)^{-2}$.

49. $(1-2\sqrt{-x})^{-1}$.

50. $(ax^{-6}+1)^{\frac{1}{3}}$.

51. $(b^3-2b^2)^{-4}$.

52. $(x-2\sqrt[3]{ax^2})^{-\frac{2}{3}}$.

53. $\sqrt[3]{a^3+\sqrt{a}}$.

54. $(1+5a^2\sqrt[3]{x})^{\frac{1}{5}}$.

55. $(ab^3-3b\sqrt{a})^{\frac{2}{3}}$.

56. $5(x+\sqrt[3]{2xy})^{-\frac{3}{4}}$.

57. $\left(\frac{1}{\sqrt{a}}-\sqrt{3x}\right)^{-2}$.

58. $\frac{1}{a-b\sqrt{c}}$.

59. $\frac{1}{\sqrt{a+2b^3}}$.

60. $\frac{2\cdot ab}{\sqrt[3]{a^{-2}-ab^2}}$.

61. $\frac{7x}{\sqrt[4]{x-2a}}$.

62. $(a^2-3\sqrt{2})^{\frac{1}{2}}$.

63. $\frac{3ax}{\sqrt[3]{a\sqrt{x}-\sqrt{-2}}}$.

64. $\frac{2y}{(x-2\sqrt{xy})^3}$.

Find only the term required:

65. The 3d term of $(a+b)^{-2}$.

66. The 4th term of $(x-y)^{-5}$.

67. The 4th term of $(1+3\sqrt{x})^{\frac{1}{2}}$.

68. The 5th term of $(x-2\sqrt{xy})^{-2}$.

69. The 5th term of $(1-\sqrt{2x})^{-3}$.

70. The 6th term of $(a+\sqrt{-2a})^{-4}$.

71. The 6th term of $(x\sqrt{y}+3y\sqrt{x})^{\frac{2}{3}}$.

72. The 5th term of $(x^{\frac{1}{2}}-3x\sqrt[3]{y})^{-3}$.

73. The 7th term of $\frac{1}{a-\sqrt[3]{ax}}$.

74. The 8th term of $\frac{3a}{\sqrt{1-4a^2b}}$.

75. The 5th term of $\sqrt[3]{(a-6\sqrt{c})^5}$.

76. The 10th term of $\dfrac{4x^2}{(x^2+2xy^3)^{-\frac{3}{2}}}$.

77. The 9th term of $(\sqrt{2}-3\sqrt{-1})^{-1}$.

78. The 8th term of $(2+\sqrt{-2})^{-\frac{1}{2}}$.

79. The term containing x^5 in $(1+x)^{-3}$.

80. The term containing x^{14} in $(x^{-2}-2x)^{-1}$.

81. The term containing $x^{-\frac{5}{2}}$ in $(\sqrt[3]{x}-4\sqrt{a})^{\frac{1}{2}}$.

82. The term containing $x^{-\frac{3}{8}}$ in $\left[\dfrac{\sqrt{2x}}{3}-\dfrac{2\sqrt{a}}{x}\right]^{\frac{1}{3}}$.

LOGARITHMS

Exercise 62

Compute by four-place logarithm tables the values of the following:

1. 55×3.86.
2. $7.81 \times 9.3 \times .568$.
3. $8.52 \times .0917 \times 63.4$.
4. $.097 \times 63.8 \times 51.14$.
5. $8.76 \times 95.32 \div 614.3$.
6. $71.3 \times 5.888 \div 43.18$.
7. $823 \times 756 \div 4320$.
8. $\dfrac{7.61 \times 53.8 \times 4.113}{27.5 \times 1.884}$.
9. $\dfrac{328 \times 57.42}{134.2 \times 3.876}$.
10. $123.5 \times 3.586 \div 976.3$.
11. $36.95 \times 438.7 \div (356.7 \times 81.44)$.
12. $\dfrac{87.63 \times 563.8 \times .0075}{27.51 \times 9832 \times .0953}$.
13. $\dfrac{91.76 \times .00385 \times 2.1176}{7.143 \times .08885 \times 11.58}$.

14. $\dfrac{75 \times .03896 \times .4427}{83 \times .008114 \times 7.003}$.

15. $\dfrac{876.4 \times 3.175 \times .6511}{8.465 \times .1973 \times 598.6}$.

16. $\dfrac{7.663 \times 85.12 \times .00681}{43.27 \times 95.16 \times .007194}$.

17. $\sqrt[3]{89.76}$.

18. $\sqrt{17 \times 29}$.

19. $\sqrt[5]{365.4}$.

20. $\sqrt{.0837}$.

21. $\sqrt[5]{.00302}$.

22. $\sqrt[3]{.00075}$.

23. $\sqrt{93 \times 2.78}$.

24. $\sqrt[3]{951 \times .037}$.

25. $\sqrt[4]{7.13 \times 41.2}$.

26. $\sqrt{10.3 \div .049}$.

27. $\sqrt[3]{361 \div 5.88}$.

28. $\sqrt[5]{8.95 \div 16.44}$.

29. $\sqrt{\dfrac{8.5 \times 78.3}{4.1 \times 19.7}}$.

30. $\sqrt[3]{\dfrac{9.02 \times 1.762}{3.117 \times .0585}}$.

31. $\sqrt[5]{\dfrac{17.44 \times .0832}{42.11 \times 8.104}}$.

32. $\sqrt[4]{\dfrac{5.382 \times 763.5}{.03871 \times 8124}}$.

33. $\sqrt{\dfrac{.03765 \times 1.448}{37.12 \times 5.718}}$.

34. $\dfrac{.4138 \times \sqrt{9.117}}{3.108 \times \sqrt[3]{.7034}}$.

35. $\dfrac{5.167 \times \sqrt{38.27}}{77.38 \times \sqrt[3]{.09034}}$.

36. $\dfrac{.7563 \times \sqrt[3]{2.087}}{.5432 \times \sqrt[4]{.8915}}$.

37. $\dfrac{853.4 \times \sqrt{.004176}}{627.1 \times \sqrt[5]{.06329}}$.

38. $\sqrt[5]{\dfrac{57.18 \times 3.876}{7.116 \times .0485}}$.

39. $\sqrt[7]{\dfrac{5.192 \times \sqrt{63.18}}{81.32 \times .0638}}$.

40. $\sqrt[3]{\dfrac{.07162 \times .00328}{1.586 \times \sqrt[4]{3.777}}}$.

41. $\sqrt{\dfrac{.0913 \times \sqrt[3]{.07652}}{1.517 \times 7.038}}$.

42. $\sqrt[3]{\dfrac{1.716 \times 873.5}{.0173 \times \sqrt{3967}}}$.

43. $\dfrac{\sqrt{750} \times (.83)^{\frac{1}{3}}}{97 \times (.0361)^{\frac{2}{3}}}$.

44. $\sqrt[3]{\dfrac{92.3 \times .08763}{.003151 \times 5\sqrt{30}}}$.

45. $\sqrt{\dfrac{9.315 \times \sqrt[3]{76.38}}{10.14 \times \sqrt{.3876}}}$.

46. $\dfrac{37.5}{583}\sqrt[6]{\dfrac{440\sqrt{.0074}}{19\sqrt[3]{600}}}$.

47. $\sqrt{8.518} \times \sqrt[3]{91.763} \times \sqrt[4]{1.998}$.

48. $\sqrt{53.34} \times \sqrt[3]{7.116} \div \sqrt{98.15}$.

49. $\sqrt{8.176 \times \sqrt{58.31} \times \sqrt{7.116}}$.

50. $\sqrt[3]{1.716 \times 8.513 \times \sqrt{75.06}}$.

51. $\sqrt{73.14 \times 80.37 \div \sqrt[3]{5768}}$.

52. $\sqrt[3]{53.11 \times \sqrt{7.852} \div 3\sqrt{77.7}}$.

53. $\sqrt[4]{3\sqrt{85} \times 2\sqrt[3]{9.16} \times 5\sqrt[5]{44}}$.

54. $\sqrt{\dfrac{876.3 \times 5.173 \times \sqrt{.008886}}{6.385 \times 711.5 \times \sqrt[3]{.01776}}}$.

55. $\sqrt{\dfrac{.07138 \times \sqrt{.00885} \times \sqrt[3]{1.762}}{.08195 \times \sqrt{.00176} \times \sqrt[3]{8.912}}}$.

56. $\sqrt{\dfrac{571.2 \times (3.817)^{\frac{1}{3}} \times (.07161)^{\frac{1}{4}}}{88.19 \times (2.716)^{\frac{1}{3}} \times (1.558)^{\frac{1}{5}}}}$.

Solve, by logarithms, the following equations:

57. $x = 19\sqrt{21}$.

58. $x = \dfrac{26}{\sqrt{11}}$.

59. $x\sqrt{10} = 95$.

60. $x^2 = 3.47$.

61. $38\,x^2 = 235$.

62. $3\sqrt{x} = 17$.

63. $11\sqrt[3]{x} = 13$.

64. $x\sqrt{5} = \sqrt[3]{50}$.

65. $97\,x^2 = \sqrt{855}$.

66. $\sqrt{17\,x} = 9.74.$

67. $\sqrt[3]{5.5\,x^2} = \sqrt{79}.$

68. $30\,x\sqrt{7} = 61\sqrt[3]{.09}.$

69. $13\sqrt[3]{15\,x} = 27\sqrt{.011}.$

70. $26\sqrt{x} = x\sqrt{33.7}.$

71. $\dfrac{17.5}{\sqrt{3\,x}} = \dfrac{\sqrt[3]{41}}{8\,x}.$

72. $\dfrac{38\sqrt[3]{x}}{97} = \dfrac{x\sqrt{55}}{79}.$

73. $\dfrac{88\,x^{\frac{2}{3}}}{9} = \sqrt{190}.$

74. $17\,x^{-\frac{1}{2}} = 65.$

75. $\dfrac{x\sqrt{87}}{75} = \dfrac{\sqrt[3]{38}}{\sqrt{x}}.$

76. $5^x = 30.$

77. $6^x = 75.$

78. $7^x = 15.$

79. $8^x = 100.$

80. $4.5^x = 50.$

81. $65^x = 3.$

82. $40^x = 5.$

83. $18.6^x = 1.86.$

84. $9^{x+1} = 15.$

85. $3^{x-1} = 36.$

86. $25^{x-1} = 11^x.$

87. $55^{2-x} = 21^{x+1}.$

88. $(3+.05)^{\frac{1}{2}x} = 100.$

89. $.9^{\frac{1}{x^2}} = 4.7^{-\frac{1}{2}}.$

90. $(1.025)^{3x} = 1.01.$

Calculate:

91. $\log_4 20.$

92. $\log_7 500.$

93. $\log_8 35.$

94. $\log_{13} 29.$

95. $\log_{16} 60.$

96. $\log_{15} 5.$

97. $\log_9 1.9.$

98. $\log_{20} .4.$

99. $\log_{2.5} .58.$

100. $\log_{12.3} .0423.$

101. $\log_{.5} .63.$

102. $\log_{.3} .07.$

103. Find the amount of $ 575 in 10 years at 5%, compound interest.

104. Find the amount of $ 8500 in 12 years at 4%, compound interest.

105. Find the amount of $ 3500 in 6 years at 6%, compound interest.

106. At what rate will $ 12,000 amount to $ 14,587 in 4 years?

107. At what rate will $8250 amount to $11,627 in 10 years?

108. What sum will amount to $520.75 in 6 years at 5%, compound interest?

109. What sum will amount to $817.30 in 8 years at 7%, compound interest?

110. In what time will $5000 amount to $8000 at 5%, compound interest?

111. In what time will $2750 amount to $5000 at 4%, compound interest?

The following examples in this exercise are to be done without the use of logarithmic tables.

112. Find the logarithm of 27 to the base 3.

113. Find the logarithm of 3 to the base 27.

Find:

114. $\log_4 8$.
115. $\log_{25} 125$.
116. $\log_{27} 81$.
117. $\log_8 32$.
118. $\log_2 \frac{1}{4}$.
119. $\log_{\frac{1}{4}} 8$.
120. $\log_{100} 10$.
121. $\log_{\frac{1}{81}} 27$.
122. $\log_{\frac{1}{27}} 81$.
123. $\log_{32} \frac{1}{16}$.
124. $\log_{64} \frac{1}{32}$.
125. $\log_{1000} .01$.

Find x, if:

126. $\log_x 8 = 3$.
127. $\log_x 81 = 4$.
128. $\log_x 125 = 3$.
129. $\log_x 6 = \frac{1}{2}$.
130. $\log_x 27 = 1\frac{1}{2}$.
131. $\log_x 32 = 1\frac{2}{3}$.
132. $\log_x 27 = -3$.
133. $\log_x 64 = -1\frac{1}{2}$.
134. $\log_x 7 = -\frac{1}{2}$.
135. $\log_x \frac{8}{27} = -1\frac{1}{2}$.
136. $\log_x 243 = -2.5$.
137. $\log_x \frac{64}{125} = -.75$.

Find x, if:

138. $\log_3 x = 4$.
139. $\log_5 x = 3$.
140. $\log_9 x = \frac{1}{2}$.
141. $\log_8 x = \frac{2}{3}$.
142. $\log_{49} x = \frac{1}{2}$.
143. $\log_{36} x = -\frac{1}{2}$.
144. $\log_{64} x = -1\frac{2}{3}$.
145. $\log_{\frac{1}{9}} x = 1.5$.
146. $\log_{27} x = -1\frac{1}{3}$.
147. $\log_{\frac{1}{4}} x = -.5$.
148. $\log_{\frac{2}{5}} x = -3$.
149. $\log_{12} x = -1.5$.

Write out as a polynomial:

150. $\log a^2x^5$.
151. $\log ab^3$.
152. $\log \sqrt[5]{cd^2}$.
153. $\log \dfrac{ax^2}{cy^3}$.
154. $\log 3\,ax^5$.
155. $\log \dfrac{5\,x\sqrt{y}}{2\,z^4}$.
156. $\log \dfrac{7\,x^2\sqrt{y}}{3\sqrt{ab}}$.
157. $\log \dfrac{11\,x^{11}}{10\,y^{10}}$.
158. $\log 4\,(x-y)^2$.
159. $\log 8\,a^3\,(a^2-b^2)$.
160. $\log 3\,a\,(9-n^2)^{\frac{1}{2}}$.
161. $\log a\,(a+b)^2(a^3-b^3)$.
162. $\log 8\,ab^3\sqrt[3]{5\,a^2b^{-1}c^{-\frac{1}{2}}}$.
163. $\log \dfrac{2\,a\,(m^2+n^2)^2}{61\sqrt{x^2-1}}$.

Change to the logarithm of a single term:

164. $\log a + 3\log b$.
165. $\log 7 - 3\log x$.
166. $\log m + \frac{1}{2}\log 7$.
167. $\log 11 + \frac{1}{3}\log a$.
168. $3\log a - 2\log b$.
169. $\log 2 + \log 3 - \log 13$.
170. $2\log 3 + 3\log a - \log 5 - \frac{1}{2}\log x$.
171. $\log 5 + 3\log x + \frac{1}{2}\log y - 2\log 7 - \frac{1}{3}\log a$.
172. $\frac{1}{2}\log 2 - \frac{1}{4}\log x + 2\log a - \frac{2}{3}\log y - \log z$.
173. $\log 7 + \frac{1}{5}\log a + \frac{3}{5}\log b - \log 6 - \frac{1}{4}\log c$.
174. $\log (a+1) + \log (a-1) - \log 2 - \frac{1}{2}\log m$.
175. $\frac{1}{2}\log (p+q) + \frac{1}{2}\log (p-q) - \frac{1}{3}\log (x+y) - \frac{1}{3}\log (x-y)$.

176. $\log 3 + 2\log 2 + \frac{1}{2}\log 5 - \log 7 - \log 11 - \frac{2}{3}\log 13.$

177. $3\log 2 + 1 + \frac{2}{5}\log 7 - \frac{1}{2}\log 3 - \frac{1}{4}\log(a^2 + b^2) - \log a.$

Find x, if:

178. $a^x = bc^2.$

179. $3b^x = cd^x.$

180. $5m^x = n^2p^{2x}.$

181. $a^{x-1} = b^3.$

182. $3a^{x-2} = d^2.$

183. $ab^x = cd^{x-1}.$

184. $m^{2x}n^{x-3} = p.$

185. $c^{3x}d = l^{x+1}m^{x-1}.$

186. If $\log 365 = 2.5623$, write $\log 3.65$. $\log .00365$.

187. If $\log 7.008 = 0.8456$, what is $\log 7008$? $\log 70.08$?

188. If $\log 27.8 = 1.4440$, write $\log 27800$. $\log .00278$.

189. If $\log 536 = 2.7292$, and $\log 537 = 2.7300$, what is $\log 5.365$? What is $\log .05368$?

190. If $\log 3.71 = 0.5694$, and $\log 37.2 = 1.5705$, what is $\log 37140$? What is $\log .003717$?

191. If $\log 709 = 2.8506$, and $\log 7100 = 3.8513$, find $\log .07096$. Find $\log 70.94$.

192. If $\log 627 = 2.7973$, and $\log 628 = 2.7980$, find x, if $\log x = 0.7975$. If $\log x = 8.7978 - 10$. If $\log x = 3.7976$.

193. If $\log 3.35 = 0.5250$, and $\log 33.6 = 1.5263$, find x, if $\log x = 9.5254 - 10$. If $\log x = 7.5260 - 10$. If $\log x = 4.5258$.

194. If $\log 2.357 = 0.37236$, $\log 235.8 = 2.37254$, and $\log x = 3.37243$, find x. Also find x, if $\log x = 7.37251 - 10$.

Given, $\log 2 = 0.3010$; find the following logarithms:

195. $\log 4$; $\log 40$; $\log 8$; $\log 800$; $\log 32$; $\log 3.2$.

196. $\log \sqrt{2}$; $\log \sqrt[3]{2}$; $\log \sqrt[3]{.2}$; $\log \sqrt[3]{4}$; $\log \sqrt{.02}$.

197. $\log 5$; $\log 50$; $\log \sqrt{5}$; $\log 2.5$; $\log 12.5$; $\log 6\frac{1}{4}$; $\log \frac{5}{8}$.

Given, $\log 2 = 0.3010$ and $\log 3 = 0.4771$; find the following logarithms:

198. $\log 6$; $\log 12$; $\log 18$; $\log 15$; $\log 150$; $\log 14.4$.

199. $\log 2\sqrt{3}$; $\log 3\sqrt{2}$; $\log \sqrt{30}$; $\log \sqrt[3]{.003}$; $\log 45$.

200. $\log 540$; $\log .024$; $\log 8\frac{1}{8}$; $\log 4\frac{1}{2}$; $\log 3\frac{3}{8}$.

201. From $\log 16$ how can one get $\log 2$? $\log 4$? $\log 8$? $\log 5$? $\log 25$?

202. From $\log 2$ and $\log 15$ how can one find $\log 3$?

203. From $\log 5$ and $\log 14$ how can one find $\log 7$?

204. From $\log 50$ and $\log 36$ how can one find $\log 3$?

205. From $\log 14$, $\log 15$, $\log 16$, how can one find the logarithms of all numbers from 1 to 10?

206. Show that there will be 31 figures in the 100th power of 2. [$\log 2 = 0.3010$.]

207. How many digits in 49^{50}? [$\log 7 = 0.8451$.]

208. Find x if $a^{x^2} = b$.

209. Find x if $a^{2x^2} = 2b$. If $2a^{2x^2} = 2^5$.

210. Find x, (a) if $.6^x = 3$, (b) if $.08^x = .9$, provided it is given that $\log 2 = 0.3010$, $\log 3 = 0.4771$.

211. What is the base if $\log .25 = -\frac{2}{3}$? If $\log \frac{1}{27} = \bar{2}.5$?

212. Solve $2^{2x-y} = 32$ and $3^{x+y} = 81$.

213. Find x and y, if $4^{x+2y} = 128$ and $25^{2y-3x} = 125$.

214. If $\log 5 = 0.6990$, find x in the equations $2^{x^2} = 40$ and $(2^x)^2 = 40$.

215. Show that $\log \frac{8}{9} + \log \frac{27}{50} - 2 \log .4 = \log 3$.

GENERAL REVIEW

Exercise 63

1. Solve the equation $12x^2 - 17x = 40$ for x.

2. Tell by inspection the sum, product, and nature of the roots of $3x^2 - 11x + 15 = 0$, and of $3x^2 + 11x = -8$.

3. Find the sum of the series $4\frac{1}{6}$, $1\frac{2}{3}$, $\frac{2}{3}$, $\cdots$, to infinity.

4. Solve for x and y, $x^3 - y^3 = 152$, and $x = 8 + y$.

5. Define quadratic equation, pure quadratic, symmetrical expression, homogeneous expression, logarithm, arithmetical progression, geometrical progression, alternation, composition, and mean proportional. Give an illustration of each.

6. Form that quadratic whose roots shall be $1\frac{1}{3}$ and $-2\frac{1}{2}$. Also that one with $\frac{5 \pm \sqrt{3}}{2}$ for roots.

7. In the A. P. 15, $13\frac{1}{2}$, 12, $\cdots$, find the 55th term and the sum of the first 20 terms.

8. Solve the equation $x^3 - 1000 = 0$ for its 3 roots.

9. Solve $2x^2 - 3xy + y^2 = 3$, $3x^2 - xy = 2$, for x and y.

10. Which term in the series $2\frac{1}{3}$, 3, $3\frac{2}{3}$, $\cdots$, is 65?

11. Find all the values of x in $x^3 = 9x$.

12. When are the roots of $lx^2 + mx + n = 0$ real? When equal? When irrational? When imaginary?

13. From the letters in the word *scholar*, how many words can be formed, of 4 letters each? Of 7 letters each?

14. Solve $\frac{1}{x^3} - 7x^{-\frac{3}{2}} = 8$. Are both the values of x roots?

15. Find the values of x and y in $x^2 - 2xy - x = 3$, $3x - 4y = 7$.

16. Find the values of x, by inspection, in

$$2ax(x^2 - 4)(3x^2 + 5) = 0.$$

17. What is the logarithm of 216 to the base 36? Of 8 to the base 128? What is $\log_{27} 2\frac{1}{43}$?

18. If $a : b = c : d$, prove that

$$a^2 + b^2 : c^2 + d^2 = 2a^2 - 3b^2 : 2c^2 - 3d^2.$$

19. Write the equation $32^{\frac{1}{2}} = 64$ in logarithmic form. What is the number? The logarithm? The base? The characteristic? The mantissa? Write $\log_{27} 243 = 1\frac{2}{3}$ as an equation.

20. How many parties of 7 each can be selected from a school of 25 girls?

21. Solve $2\sqrt{x+3} - \sqrt{x-2} = \sqrt{3x-2}$. Are both values of x roots of this equation? Why?

22. Find the 6th term and the sum of 7 terms in the series $-4, 6, -9, \cdots$.

23. Solve $\frac{2-x}{3-x} - 2 - \frac{x-4}{x+2} = 0$.

24. Prove the formula for sum of a G. P. if the first term, last term, and ratio are given. Derive formula for sum of infinite geometrical series.

25. Solve $a(6a - 13) - 2ax(a - x) = 5(x + 3)$.

26. Why cannot a negative number have any logarithm provided the base is always positive? How are operations with negative numbers performed by aid of logarithms?

27. Expand $(2\sqrt{x} - x^2\sqrt[3]{y})^5$.

28. Find the values of a that will make the equation $4x^2 - 15x - ax + a + 20 = 0$ have equal roots. Prove your values correct.

29. In an A. P., $l = 14$, $n = 40$, $s = 430$; find a and d.

30. Solve $2\sqrt{x-2} + \sqrt{2x+3} = \sqrt{8x+1}$.

31. From the letters in the word *sweetest*, how many arrangements can be made, taking all the letters every time?

32. Find the middle term of $(x^2y - \frac{1}{2}\sqrt{x^{-3}})^{10}$.

33. Solve $x^2 + xy - y^2 = 1$, $xy + 2y^2 = 3$.

34. Insert 5 geometrical means between $2\frac{2}{3}$ and $30\frac{3}{8}$.

35. Prove the formulas for l in the progressions. Also prove the formulas for sum in arithmetical progression.

36. Solve for x, $2\sqrt{3x-2} - 3\sqrt[4]{3x-2} = 2$.

37. Compute by logarithms, $\dfrac{\sqrt{37.5 \times 9.863}}{\sqrt[3]{.0716 \times 438.6}}$.

38. Form the quadratic whose roots will be $-a$ and $\dfrac{a-1}{2}$. The quadratic whose roots are $\dfrac{5 \pm 2\sqrt{-1}}{3}$.

39. Distinguish between mean proportional and third proportional. Find the mean and the third proportional to a and $2ab$.

40. From a class of 8 boys and 10 girls, how many groups of 2 boys and 3 girls each can be selected?

41. Solve $\dfrac{2x-3}{3x+4} - 3 = -\dfrac{x+17}{6x+3}$.

42. Insert 23 arithmetical means between 3 and 63. Also 35 means between $\frac{1}{2}$ and 27.5.

43. Solve $x + 2y = 2$, $x^3 + 8y^3 = 26$.

44. Solve $(x^2+x-5)^2-8(x^2+x)+47=0$.

45. From 16 consonants and 4 vowels, how many words can be formed, each consisting of 3 consonants and 2 vowels?

46. Compute by logarithms the value of $\sqrt[3]{\frac{.8034\times\sqrt{.05876}}{.5138\times\sqrt[4]{.00175}}}$.

47. A man agreed to dig a well at the rate of 25 cents for the first yard, 50 cents for the second, 75 cents for the third, and so on. Upon completion he received \$30. How deep was the well?

48. Find the value, by logarithms, of $2^{\frac{1}{2}}\times(\frac{1}{2})^{\frac{2}{3}}\times\sqrt[3]{\frac{3}{2}}\times\sqrt{.1}$.

49. What is the value of $(x-\sqrt{2})^4+(x+\sqrt{2})^4$?

50. How many figures in 65^{65}?

51. Solve $x^2-xy+2y=4,\ 2x+3y=5a+1$.

52. If $a:b=c:d$, prove $a^2+4b^2:c^2+4d^2=ab:cd$.

53. Prove the formulas for the number of permutations and the number of combinations of m things taken n at a time.

54. Form the quadratic whose roots are $0, -\frac{3a}{4}$. Also the one whose roots will be $\frac{-1\pm4\sqrt{-3}}{7}$.

55. Solve $(2x-3)^2-(x-1)^2=5$.

56. Express as a polynomial, $\log\frac{1}{2}a^2\sqrt{b}$; $\log\frac{1}{3}x^3\sqrt[3]{3y}$.

57. Compute by logarithms, $\frac{83}{101}\sqrt[3]{\frac{.071}{.809}}$.

58. Simplify $(\sqrt{1-x^2}+1)^3-(\sqrt{1-x^2}-1)^3$.

59. How many combinations, each containing 6 white balls and 5 red balls, can be selected from 14 white and 10 red balls?

60. Solve $2x^2+3xy-5y^2=4,\ 2xy+3y^2=-3$.

61. Find x that will make $2x-1$, $x+7$, $3x+1$, in A. P. Find x if they are in G. P.

62. Find s that will make the roots of $3sx^2+2sx+9x+8=0$ equal to each other.

63. If $a:b=c:d$, prove

$$\sqrt{a^2+b^2}:\sqrt{c^2+d^2}=\sqrt[3]{a^3-b^3}:\sqrt[3]{c^3-d^3}.$$

64. Solve $8x^{-3}+11\sqrt{x^{-3}}=54$.

65. Twenty men are going to march four abreast. In how many ways can they place themselves?

66. Expand $(\sqrt{2a}-\sqrt{x})^6$.

67. Solve $y-\frac{1}{x}=x-\frac{1}{y}=\frac{3}{2}$.

68. Solve $2x^2-ax-a^2=2(3x-2)$, for x.

69. Find, by logarithms, the value of x, if $7^x=100$.

70. Without solving the equations, tell what is the sum, product, and character of their roots: (*a*) $3x^2-7x=-2$; (*b*) $5x^2+4x+1=0$.

71. What is the value of $2.7181818\cdots$?

72. Solve $2x^2-5x-\sqrt{2x^2-5x+1}=1$.

73. Form that quadratic the sum of whose roots is 11 and whose product is $13\frac{1}{2}$. Prove your answer.

74. If $a:b=b:c=c:d$, prove that $a+b:b+c=2a-b:2b-c$.

75. Solve $abx^2-a^2(x-1)=b^2(x+1)$.

76. Find all the values of x and y in the equations $(x+y)^2-x-y=6$, $x^2y^2+7xy+12=0$.

77. The sum of 35 terms of an A. P. is 490 and the common difference is $\frac{3}{4}$. Find the first and last terms.

78. If m and n are the roots of $x^2 - x + 1 = 0$, show that $m^2 + n^2 + 1 = 0$. Show that $\frac{1}{m} + \frac{1}{n} = 1$. That $m - n = \sqrt{-3}$.

79. The sum of five terms of a G. P. is $5\frac{31}{81}$ and the ratio is $\frac{2}{3}$. Find the first and last terms.

80. Find the four roots of $8x^4 = 27x$.

81. Prove that the roots of $ax^2 + 2bx + c = 0$ will be equal if b is a mean proportional between a and c. When will the roots be rational?

82. Expand $(1 - 4x)^{-\frac{1}{2}}$ to 5 terms.

83. Find the limit of the sum of $1 - \frac{2}{5} + \frac{4}{25} \cdots$ to infinity.

84. Solve $4x + 4\sqrt{3x^2 - 7x + 3} = 3x^2 - 3x + 6$.

85. Find two numbers whose difference, sum, and product are to each other as 2 : 3 : 5.

86. There are three numbers in A. P. whose sum is 3. If 3, 4, and 21 be added to them, respectively, the sums form a G. P. Find the numbers.

87. How many terms of the series 32, 48, 72, ···, amount to 665?

88. Form the equation whose roots are double the roots of $x^2 - 3x + 2\frac{1}{2} = 0$.

89. The 5th term of an A. P. is -3 and the 15th term is 17. Find the sum of the first 20 terms.

90. How many arrangements can be made from the letters in the word *holiday*, taken all together? How many, if the three letters *lid* are never separated?

91. If the base of a system of logarithms be 4, tell the logarithm of each: 16; 8; 32; 2; $\frac{1}{4}$; 1; $\frac{1}{8}$; $\frac{1}{2}$; $\frac{1}{32}$; $\sqrt{\frac{1}{2}}$; $\sqrt[3]{\frac{1}{4}}$; $\sqrt[7]{\frac{1}{256}}$.

92. Solve $\frac{1}{x}+\frac{1}{y}=1,\ \frac{1}{x^2}+\frac{1}{y^2}=13.$

93. How many terms of the series $4\frac{1}{3}$, 4, $3\frac{2}{3}$, $\cdots$, amount to 21?

94. Find that G. P. the sum of an infinite number of whose terms is 4 and the second term is $\frac{3}{4}$.

95. Compute the 5th root of $\frac{4.281\times\sqrt{.09176}}{321.7\times\sqrt[3]{.008074}}$ by logarithms.

96. Find the 10th term of $\frac{\sqrt{ax}}{(1-2\sqrt{ax})^2}$ by the binomial theorem.

97. Find all values of x in $1+2x^2+3x=3\sqrt{2x^2+3x-1}$.

98. If a, b, c, d, are in continued proportion, prove that

$$la^2+mb^2+nc^2 : lb^2+mc^2+nd^2 = ad+bc : 2cd.$$

99. Solve this equation for the value of x: $x+\frac{b}{a}=\frac{1}{x}+\frac{a}{b}$.

100. Find x and y in $x^2+y^2=3\frac{1}{4},\ x^{-2}+y^{-2}=1\frac{4}{9}$.

101. Find the sum of all numbers between 10 and 500 exactly divisible by 7.

102. What is meant by "completing the square"? How is it done? What is an imaginary? What are conjugate imaginaries? Prove that $\log PQ=\log P+\log Q$.

103. From the usual formulas for l and s of an A.P. derive a formula for a not containing s. A formula for l not containing a. A formula for d not containing l.

104. Out of 15 consonants and 5 vowels how many words can be made each consisting of 4 consonants and 3 vowels?

105. Find, to 3 decimal places, the logarithm of 65 in a system whose base is 15.

106. Solve $\sqrt{x+y}=\sqrt{y}+2,\ x-y=7.$

107. When are 3 quantities in continued proportion? Prove that if a, b, c, are in continued proportion, then a, $a+b$, $a+2b+c$, are also in continued proportion.

108. Find x if $(x^2+2x)^2-18x(x+2)+45=0.$

109. If the base of a system of logarithms is a, what is $\log a$? $\log \frac{1}{a}$? $\log a^3$? $\log \sqrt{a}$? $\log \sqrt[5]{a^4}$? $\log \sqrt[n]{\frac{1}{a^m}}$?

110. Solve $\frac{1}{x}-\frac{1}{y}=2,\ \frac{1}{x^3}-\frac{1}{y^3}=3\frac{1}{2}.$

111. Calculate by logarithms the mean proportional between $\sqrt{5.082}$ and $\sqrt[3]{.009116}$.

112. In a G. P. the 5th term is 12 and the 11th term is 768. Find the 3d term and sum of 9 terms.

113. Solve $\frac{1}{a+b-x}=\frac{1}{a}-\frac{1}{x}+\frac{1}{b}.$

114. Find x if $2.5^x=75.$

115. How many numbers, of 6 different digits each, can be written from our 9 significant digits?

116. Find x and y if $x^2-xy=1\frac{1}{2}$ and $xy+y^2=1.$

117. Insert 5 geometrical means between 5 and 3645. Also 69 arithmetical means between 5 and 3645.

118. Compute the value of $\left(\frac{-4.116\times 75.38\times .0567}{31.24\times(-1.909)\times .0053}\right)^{\frac{2}{3}}.$

119. Solve for x and y: $x^2+xy+y^2=13;\ x^4+x^2y^2+y^4=91.$

120. Find x and y in $x^2+y^2+x+y=26;\ xy=-10.$

121. Solve for x: $\frac{x-1}{x-3}-1-\frac{3x+2}{x+3}=0.$

122. Given $\log 40 = 1.60206$, find $\log 2$; $\log 5$; $\log 20$; $\log 50$; $\log \sqrt{5}$; $\log \sqrt{2.5}$; $\log \frac{1}{2}$; $\log \frac{1}{4}$; $\log \frac{2}{5}$; $\log 1.25$; $\log 2\frac{1}{2}$.

123. If $s : t = u : v = w : x = y : z$, prove that

$$s + u + w + y : t + v + x + z = s : t = \text{etc.}$$

124. Find all the values of x in: $3\,a^2x\,(x+1)\,(x^4-81) = 0$.

125. Solve for x and y: $x^2 + y^2 + x - y = 26$; $xy + 15 = 0$.

126. From a delegation of 15 Protestants and 11 Catholics, there is to be chosen a committee of 6 Protestants and 4 Catholics. In how many ways might this be accomplished?

127. If $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 9$ and $x + y = 41$, find x and y.

128. If $\sqrt{3\,x-2} + \sqrt{x+2} - 4 = 0$, find x and discuss its values as roots.

129. Given $a = 2$, $l = \frac{1}{3^{\frac{1}{2}}}$, $s = \frac{1^{\frac{1}{2}}}{3^{\frac{1}{2}}}$, find r and n.

130. Expand $(2 - \sqrt{-2})^5$.

131. Solve $x^{\frac{1}{3}} - y^{\frac{1}{3}} = 7$, $x - y = 133$.

132. The 5th term of a G. P. is 336 and the 9th term is 5376. What is the 2d term?

133. Find x: $\dfrac{x + \sqrt{12\,a - x}}{x - \sqrt{12\,a - x}} = \dfrac{\sqrt{a}+1}{\sqrt{a}-1}$. Discuss its values.

134. If a certain number is divided by 8, the result will be the same as if 16 were divided by the number and $3\frac{1}{2}$ added to the quotient. What is the number?

135. If $v : x = x : y = y : z$, prove that $x + y$ is a mean proportional between $v + x$ and $y + z$.

136. If $x + y + 1 = x^2 - y^2 + xy - 1 = 0$, find x and y.

137. What are the values of x in the equation $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$? Are these values both roots?

138. Solve $x^2-2ax-2bx+(a+b+c)(a+b-c)=0$.

139. The sum of the first two terms of a G. P. is 72 and the sum of the next two is 8. What is the 1st term? The 5th term?

140. Expand to 4 terms: $(y^{-\frac{2}{3}}-5x^2\sqrt{-1})^{-\frac{2}{5}}$.

141. Find the series in which $d=8$, $l=147$, and $s=1425$.

142. If the ratios $l:m$, $n:p$, $q:r$, are equal, prove that each is equal to $\sqrt{\frac{l^2+n^2+q^2}{m^2+p^2+r^2}}$.

143. Solve $(3x-2)^2+(x-\frac{2}{3})=84$.

144. Prove the binomial formula for positive integral exponents.

145. What is the 7th term of $(x^{-1}-\frac{1}{2}\sqrt{xy^3})^{20}$?

146. Solve $x^2+xy+2y^2=11=x^2+\frac{5}{2}y^2$.

147. Find x if $2^{x^2}=500$.

148. From the figures 1, 2, 3, 4, 5, 6, 7, how many numbers can be formed of 5 different figures each? How many of these will contain a 3? How many will have 3 and 6 together? How many will be odd?

149. Multiply $.03716^{\frac{1}{3}}$ by $1.8716^{\frac{2}{5}}$ by logarithms.

150. If, in an A. P., $a=s=-\frac{5}{3}$ and $n=20$, find d and l.

151. Find that G. P. whose sum to infinity is $1\frac{1}{3}$ and whose 2d term is $\frac{1}{3}$.

152. Solve $\frac{x^2}{m+n}-\left(1+\frac{1}{mn}\right)x+\frac{1}{m}+\frac{1}{n}=0$.

153. Solve $\frac{1}{x}+\frac{1}{y}=\frac{5}{18}$, $xy=54$.

154. Insert 6 geometrical means between 5 and -640.

155. Find x and y if $3^x 5^y = 75$, $2^x 7^y = 98$.

156. How many different sounds can be made by striking 16 keys of a piano, 3 at a time?

157. If $2(\sqrt{x}-3)^2-3=\sqrt{x}$, find x.

158. Expand $(2\sqrt{3}+3\sqrt{2})^4$.

159. Find x and y from $x+y=8$, $x^2y^2-28xy=-192$.

160. Solve $6x^2-3x=2+\sqrt{2x^2-x}$.

161. There are two fractions the sum of whose denominators is 5. The numerator of the first is the square of the denominator of the second, and the numerator of the second is the square of the denominator of the first. The sum of the fractions is $5\frac{5}{6}$. Find them.

162. The sum of an infinite number of terms of a certain G. P. is $4\frac{1}{6}$. The sum of the first two terms is $2\frac{2}{3}$. Find the series.

163. If $\log 13 = 1.1139$, what is $\log \sqrt[3]{1300}$? $\log \sqrt{.0013}$?

164. Find x: $\sqrt{x+1} : \sqrt{2x} = \frac{1}{x} : \frac{2}{x+4}$.

165. Evaluate the decimal $1.4363636\cdots$.

166. Solve $9x-3x^2-4\sqrt{x^2-3x+5}=0$.

167. Out of 7 consonants and 4 vowels how many words can be formed, each consisting of 3 consonants and 2 vowels?

168. Solve for x: $(x^{-2}+\frac{1}{4})^{-3}=27$ and $(x^{-\frac{1}{3}}-\frac{1}{3})^{-\frac{1}{3}}=-\frac{1}{3}$.

169. Find the numerical value of

$$\tfrac{1}{2}\log_3 9 - 2\log_{27} 3 + \log_a a^2 - \log_a 1.$$

170. Show that $ma+nb : pa+qb = mc+nd : pc+qd$ provided $a, b, c, d,$ are proportional.

171. Find x if $4^{3x-1} = (\frac{1}{2})^{x-5}$. [Without tables.]

172. How many games must be played in a league of 10 baseball clubs, provided each club plays 10 games with every other club?

173. Insert 5 arithmetical means between -7 and 77.

174. Find the values of x, correct to two decimal places, in the equation $x^2 - 2x - 2 = 0$.

175. Solve $x^2 + 5xy + 3y^2 = 3$, $3x^2 + 7xy + 4y^2 = 5$.

176. If the series 12, 9, $\cdots$, is arithmetical, find the sum of 20 terms. If geometrical, find the sum of an infinite number of terms.

177. Solve $x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1 = 0$.

178. Tell sum, product, and nature of roots of $3x^2 - 4x - 11 = 0$. Also of $2x^2 + 3x + 7 = 0$.

179. Find the sum of all the odd numbers between 20 and 220.

180. Expand $(\frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{-2})^3$ and simplify.

181. If $a, b, c, d,$ are proportional, show that $ab + cd$ is a mean proportional between $a^2 + c^2$ and $b^2 + d^2$.

182. Solve the equations $\dfrac{4}{x} + \dfrac{7}{y} = \dfrac{8a}{xy}$, $x^2 + y^2 = a^2$.

183. Find two numbers in ratio of $7 : 5$, the difference between whose squares is 96.

184. Find the 8th term of the expansion of $\sqrt[3]{a-6\sqrt{ax}}$, by the binomial theorem.

185. Solve for y: $1+\sqrt[3]{8y^3-3y^2}=2y$.

186. Solve $x+y+2\sqrt{x+y}=24$, $x-y+3\sqrt{x-y}=10$.

187. Find the value of z: $a^2z-2b^2=ab\dfrac{z^2+1}{z+1}$.

188. Solve $\dfrac{2}{x}+\dfrac{3}{y}=5$, $\dfrac{5}{x^2}+\dfrac{2}{xy}-\dfrac{3}{y^2}=-19$.

[HINT: Let $\dfrac{1}{x}=m$, $\dfrac{1}{y}=n$.]

189. Discuss the values of x in the equation,

$$\sqrt{3x+1}=\sqrt{9x+4}-\sqrt{2x-1}.$$

190. Find, by logarithms, the values of x and y, if $3^x=50y$ and $2^x=5y$.

191. Find the $(r+1)$th term of $(1-x)^{25}$.

192. Simplify $(2-\sqrt{1-x})^4+(2+\sqrt{1-x})^4$.

193. Compute, by logarithms, the value of

$$\sqrt[3]{.07164}\div\sqrt[5]{\frac{2\sqrt{3}}{\sqrt[3]{13}}}.$$

194. Solve $\sqrt{a-x}+\sqrt{-(a^2+ax)}=\dfrac{a}{\sqrt{a-x}}$.

195. If a, b, and c are in G. P., show that $\dfrac{a}{a+b}$, $\dfrac{1}{2}$, and $\dfrac{c}{b+c}$ are in A. P.

196. Find the 3 cube roots of unity; *i.e.* solve the equation $x^3=1$.

197. Solve for x: $\frac{1}{2}\left\{\frac{1-x}{1-a}+\frac{a}{x}\right\}=\frac{a^2x+a-x+1}{x(a^2-1)}$.

198. Solve $\sqrt{5x+1}+2\sqrt{x-2}=3\sqrt{2(x-1)}$.

199. Solve $(m+x)^{\frac{1}{2}}-(m-x)^{\frac{1}{2}}=x$.

200. Of how many terms does an A. P. consist in which $d=3$, $l=302$, $a=5$?

201. The sum of an infinite number of terms of a G. P. is 15 and the second term is $3\frac{1}{5}$. Find the fourth term.

202. Solve $\frac{x^2+1}{x^2-1}-\frac{x^2-1}{x^2+1}=\frac{9}{20}$.

203. Expand $(\sqrt{2x}-\sqrt[3]{3x})^4$.

204. The difference between two numbers is 32 and the arithmetical mean exceeds the geometrical mean by 4. Find the numbers.

205. By the principles of proportion find x if

$$\frac{\sqrt{x+2}-\sqrt{x-3}}{\sqrt{x+2}+\sqrt{x-3}}=\frac{\sqrt{5x+1}-4\sqrt{x-6}}{\sqrt{5x+1}+4\sqrt{x-6}}.$$

206. Find sum of $\frac{1}{2}-1-\frac{5}{2}\cdots$ to 29 terms.

207. Find sum of $\frac{1}{2}+\frac{1}{3}+\frac{2}{9}\cdots$ to 6 terms.

208. Solve $a^2x^2-x^2-a^2x-5ax=6a^2$.

209. The sum of 10 terms of an A. P. is 100 and the sixth term is 11. Find the second term.

210. Solve $x^2-x+1+\frac{1}{x^2-x+1}=\frac{10}{3}$.

211. The sides of a right triangle are in A. P. Prove that they are proportional to 3, 4, 5.

212. Find x if

$$\sqrt{b}+\sqrt{a-x^2}:\sqrt{b}-\sqrt{a-x^2}=\sqrt{a}+\sqrt{b-x^2}:\sqrt{a}-\sqrt{b-x^2}.$$

213. How many terms of the series $1\frac{1}{3}$, 1, $\frac{2}{3}$, $\cdots$, must be taken, that the sum may be zero?

214. Solve $2x^2-3x+4=\dfrac{3}{2x^2-3x+2}$.

215. In an A. P. $a=7$, $d=-\frac{1}{3}$, $s=55$. Find n and l.

216. If $\sqrt{a+x}:\sqrt{a}+\sqrt{a+x}=\sqrt{a-x}:\sqrt{a}-\sqrt{a-x}$, find x.

Solve the following simultaneous quadratic equations and associate the corresponding values of x and y:

217. $y^2+xy+y=-6$; $x^2+xy+x=8$.

218. $x(x+y)=10$; $y(y-x)=3$.

219. $\sqrt{x+y}-\sqrt{x-y}=2$; $3x-2y=7$.

220. $x^3-\dfrac{1}{y^3}=316$; $x-\dfrac{1}{y}=4$.

221. $x^2+2xy=-21$; $y^2+2xy=-5$.

222. $x^2+y^2-4a\sqrt{x^2+y^2}=5a^2$; $x^2-y^2=7a^2$.

223. $a(x-a)=b(y-b)$; $ax+by=xy$.

224. $6x^2y^2+5xy=6$; $x+30y=12$.

225. $\begin{cases}(x-3y)^2-8x+24y=-12;\\ 2(2x+y)^2-22x-11y=-5.\end{cases}$

226. $x-y-\sqrt{x-y}=2;\ x^3-y^3=2044.$

227. $2x^2-xy=12;\ x^2-2xy+3y^2=9.$

228. $4x^2+5xy-3x-y+2=0;\ 2x+5y=-4.$

---

229. One root of the equation $x^2-6x+29=0$ is $3-2\sqrt{-5}$. Without solving this quadratic, find the other root and prove your answer correct.

230. Solve: $\dfrac{x}{x-1}-\dfrac{x}{x+1}=\dfrac{2a-2}{a^2-2a}$.

231. Find the limit of the sum of the series $5-3+1-\frac{1}{3}+\frac{1}{9}-\cdots$ to infinity.

232. Solve: $\sqrt{3x^2-2x+4}=3x^2-2x-8$.

233. If w, x, y, and z are proportional, show that $\sqrt{w^2-y^2}$, $\sqrt{x^2-z^2}$, $w+y$, $x+z$, are also proportional.

234. Find the ninth term of $(\frac{1}{2}x^2y-2\sqrt[3]{x})^{12}$.

235. Divide $9\frac{1}{2}$ into three parts in G. P. such that the sum of the first two is to the sum of the last two as 3 to 2.

236. For what values of m will the equation $4x^2-15x+36=m(x-1)$ have equal roots? Verify your results.

No.	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

No.	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Phillips and Fisher's Geometry

By ANDREW W. PHILLIPS, Ph.D.
and IRVING FISHER, Ph.D.
Yale University

PHILLIPS AND FISHER'S ELEMENTS OF PLANE AND SOLID GEOMETRY $1.75
PHILLIPS AND FISHER'S PLANE AND SOLID GEOMETRY. Abridged $1.25
PHILLIPS AND FISHER'S PLANE GEOMETRY—Separate . 80 cents
PHILLIPS AND FISHER'S GEOMETRY OF SPACE—Separate $1.25
PHILLIPS AND FISHER'S LOGARITHMS OF NUMBERS . 30 cents

The publication of this text-book marks a *new* era in the teaching of Geometry. Its distinctive qualities are: (1) clearness of presentation, both in form of statement and in the diagrams; (2) natural and symmetrical methods of proof; (3) abundance and variety of original problems for demonstration and for numerical computation.

But the feature which more than any other distinguishes it from similar text-books is the use of photo-engravings of geometrical figures arranged side by side with skeleton drawings of the same, whereby the most magnificent collection of geometrical models ever constructed is brought within reach of every preparatory school and college student. By this method of illustration the great problem of educating the student's imagination to a proper comprehension of the figures of solid geometry is practically solved.

The Abridged Edition is intended for those schools which desire a briefer course than that offered in the complete work. It has all the excellencies and features of the larger book, including the reproductions of the models.

Copies of any of these books sent, prepaid, on receipt of price.

American Book Company

New York • Cincinnati • Chicago

Lessons in Physical Geography

By CHARLES R. DRYER, M.A., F.G.S.A.
Professor of Geography in the Indiana State Normal School

Half leather, 12mo. Illustrated. 430 pages. . . . Price, $1.20

EASY AS WELL AS FULL AND ACCURATE

One of the chief merits of this text-book is that it is simpler than any other complete and accurate treatise on the subject now before the public. The treatment, although specially adapted for the high school course, is easily within the comprehension of pupils in the upper grade of the grammar school.

TREATMENT BY TYPE FORMS

The physical features of the earth are grouped according to their causal relations and their functions. The characteristics of each group are presented by means of a typical example which is described in unusual detail, so that the pupil has a relatively minute knowledge of the type form.

INDUCTIVE GENERALIZATIONS

Only after the detailed discussion of a type form has given the pupil a clear and vivid concept of that form are explanations and general principles introduced. Generalizations developed thus inductively rest upon an adequate foundation in the mind of the pupil, and hence cannot appear to him mere formulae of words, as is too often the case.

REALISTIC EXERCISES

Throughout the book are many realistic exercises which include both field and laboratory work. In the field, the student is taught to observe those physiographic forces which may be acting, even on a small scale, in his own immediate vicinity. Appendices (with illustrations) give full instructions as to laboratory material and appliances for observation and for teaching.

SPECIAL ATTENTION TO SUBJECTS OF HUMAN INTEREST

While due prominence is given to recent developments in the study, this does not exclude any link in the chain which connects the face of the earth with man. The chapters upon life contain a fuller and more adequate treatment of the controls exerted by geographical conditions upon plants, animals, and man than has been given in any other similar book.

MAPS AND ILLUSTRATIONS

The book is profusely illustrated by more than 350 maps, diagrams, and reproductions of photographs, but illustrations have been used only where they afford real aid in the elucidation of the text.

Copies sent, prepaid, on receipt of price.

American Book Company

New York • Cincinnati • Chicago

THORNTON SMITH
2064 DUNWOODY ST.

THORNTON SMITH
2054 DUMWOODY ST.